THE

MISCELLANEOUS

MATHEMATICAL PAPERS

OF

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PREFACE.

THE objects of this work are, to gather into one volume the scattered Miscellaneous Papers of OLIVER BYRNE, to correct errors of inadvertence, and those of the press, chiefly introduced through incidents that attend periodical publications. The papers in my possession, collected during the last thirteen years, will make about ten numbers, each the size of the present, to be published quarterly; and I promise with confidence, from existing arrangements, that their publication will be continued independent of their immediate pecuniary returns. As the articles issue this second time from the press, the points that some journalists and reviewers have apparently misunderstood, when necessarily condensed, shall be rendered more comprehensive.

JOHN BYRNE.

LONDON,
October 1, 1848.

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MATHEMATICAL MISCELLANY.

A NEW THEORY OF THE EARTH, THAT FULLY ACCOUNTS FOR MANY ASTRONOMICAL, GEOGRAPHICAL, AND GEOLOGICAL PHENOMENA, HITHERTO UNACCOUNTED FOR.

The following particulars of this interesting theory were gathered from papers that appeared in the "Polytechnic Journal" for 1843; and from lectures delivered by Professor Oliver Byrne, at the Museum Lecture Room, Philadelphia, in January, 1842, and at the Manchester Mechanics Institution in December, 1843.

THE limited life and understanding of man have caused most of his rules to admit of exceptions, and by the same agents almost all his discoveries respecting the laws of nature are embarrassed with apparent irregularities. However, by patient investigation, and accurate observations from time to time, laws yet more general are being continually advanced, taking in former theories and their exceptions; but exceptions will be found while such theories remain in any way at variance with truth or nature.

Nothing more clearly shows the ultimate perfectibility of man than this gradual advance, this approximate approach to perfection; and if it were necessary to give a proof, an argument more striking than this could not be advanced, to show that man possesses an immortal soul. The government of all the mighty works of nature by a few unerring and general laws, leads us to ask the question, who so wisely framed them? The true answer in our time is almost on the lips of every one: an All-wise and All-powerful God.

From the unwillingness of man to investigate, and from his proneness to copy, improvements have been slow, and therefore the generalization of limited theories and theorems, things of rare occurrence. Even in the exact sciences of mathematics, and in those depending upon mathematical investigation, we find oversights, logical objections, and even errors, transmitted from age to age by those who are termed learned. Aye, in mathematics, the sanctuary and stronghold of absolute certainty, which claims exception from the infirmities that are attached to all other branches of human knowledge, we find such men as Newton, Laplace, Emerson, Euler, Tom. Simpson, Gregory, and others, to differ in opinion, either from taking different views of subjects, or from the limited state of knowledge in the times they flourished. Certainly differences on mathematical

points are of very rare occurrence, for although individuals may here commit mistakes, or differ respecting certain mathematical subjects, such oversights can only remain undetected for a short time; as the study of mathematics is a self-correcting one, based on sure, simple, and evident principles, which have no relation to the interest or passions that influence the mind. Therefore any theory or proposition, which is capable of being submitted to mathematical investigation, must ultimately be adjusted, no matter how loosely it may be presented at any time, or how its investigators may have erred. It may further be advanced with respect to mathematical investigation. Although the sciences of mathematics are coeval with man, and have been cultivated with the greatest avidity by the greatest minds of every age, in every civilized nation; although their extent and application are at present very great, considered by some to be capable of little further advance, yet it may safely be asserted, that they are only in their infancy: as long as man continues to improve, so long will the bounds of mathematics continue to extend, till all other human inquiries become subject to its simple and unerring principles.

The theory which is here promulgated, and which we shall endeavour to exemplify and explain in the simplest terms possible, is capable of being submitted to the most exact and rigorous mathematical scrutiny. Yet in this place we prefer establishing it by a general concurrence of facts, which are known to almost every observer, rather than by an abstruse and elaborate mathematical process; because by proceeding on the former plan the subject will be understood by the many, while the latter, which is given in the proposer's new work * on the Calculus, about to be published, would only be understood by the few, who at present know enough of the uncertainty and dissatisfaction which have attended former attempts to establish the point in view by such a procedure.

By observing the apparent motions of the fixed stars and of the sun and planets, the true motions of the bodies in our solar system were discovered; not before the attention of man was for a considerable time engaged by their appearances and changes, and many theories respecting them advanced and confuted: but this, like other subjects capable of being submitted to mathematical investigation, was ultimately set right. The motions of the earth on its axis and round the sun were discovered in the same manner, by observing the apparent motions of the fixed stars. Seeing that all the stars rise and set in the course of a day, the stars must move round the earth, or the earth must revolve on an axis in that time: the truth of the latter motion was finally established. It was also observed, that the stars which appeared to set with, or immediately after,

* See the list of Books given at the end of this number.

the sun, gained an advance of him till they were lost in his rays, then appeared to pass him, and return to their former position with respect to the sun, in the course of a year. This fact shows, either that the stars move round the sun, which stood still, while the earth with revolving on its axis would possess a wabbling motion, or what might be called at the present day a great nutation, to effect the change of the seasons ; or that the earth stood in the same position revolving on its axis, while the sun made a circuit of the heavens in the course of a year ; or lastly, which was ultimately found to be true, that the sun nearly remained in the same relative position as well as the fixed stars, and that the earth moved round him in the course of a year, and that also in such a manner, that the changes of the seasons were produced. Here it would be a useless, as well as a laborious task, to give even an outline of the several theories and conflicting opinions which have prevailed before the true theory of the solar system was established. We are sorry that our present limits will not permit us to give such an outline, as it might at the same time give a caution to many not to condemn, censure, or approve, before they have investigated and understood. This theory of the earth, which likewise shows the cause of many astronomical phenomena, is not introduced and promulgated for the purpose of confuting any of the well-established laws of the solar system. The great difference between what is here advanced and other theories, is that only two of the motions of the earth are admitted, namely, the annual and diurnal. The other apparent or imaginary motions known by the terms "The precession of the equinoxes," "Solar and lunar nutations," and "The decrease in the obliquity of the ecliptic, or rather, the collapsing of the planes of the equator and ecliptic," are rejected, and the true cause of such apparent motions substituted in their stead, which we shall hereafter term *the right motion or change of the earth's axis.*

Many, as well as those who propose anything new, be it ever so true, must be well aware of the fact, that the proposer or inventor of any new scheme, be it ever so important or useful, no matter how willing and well prepared he may be to verify his claims, even in these enlightened times, frequently struggles in vain to gain attention, much less to obtain belief ; facts, it is true, are stubborn things, but prejudices are far more stubborn. This difficulty mainly arises from the confidence of mankind having been so often deceived and grossly abused by imaginary improvers and visionary inventors. However, the inventor or proposer of a new theory who can establish his claims by strict mathematical argumenfs, no matter how uncommon they may at first appear, places beyond all doubt their certainty ; and

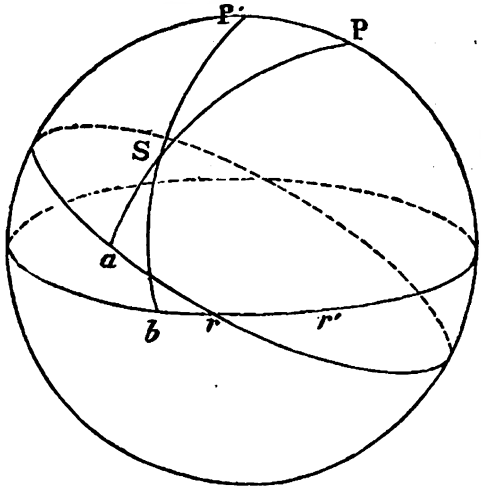
although he cannot induce people to study or think for themselves, yet he defies all efforts to confute one single tittle of anything which may be thus established. Before proceeding further, we beg to acquaint general readers, that few technicalities are used, and those retained are explained in the simplest manner possible; this remark was considered necessary, because subjects of the like nature are too often unnecessarily encumbered with technical terms. Although this theory of the changes, nature, and form of the earth is here established without the use of x's, y's, Greek characters, or many technical terms, the reasoning employed is strictly mathematical: not that we object to the mathematical plan of proceeding, but that the subject, as we have before observed, may be generally understood. To accommodate the profound mathematician as well as the general reader, this theory is established in the work before alluded to. Returning to the discussion of the principal feature of this theory, which is, substituting for what is called "The precession of the equinoxes," "Solar and lunar nutations," and "The collapsing of the planes of the equator and ecliptic," one motion, here termed, *the right motion of the earth's axis.* In the first place, we shall define or rather explain the meaning of these terms. The points where the planes of the equator and ecliptic intersect are called the equinoctial points; they have a retrograde motion, called the precession of the equinoxes. This apparent motion was observed long before the Christian era, it could not remain for any long time undetected, as the latitudes and longitudes, as well as the right ascensions and declinations of the stars, were reckoned from one of those points,—Aries. The declinations and right ascensions of the stars are reckoned in a manner similar to the latitudes and longitudes of places on the earth, only the first meridian is supposed to pass through the equinoctial points; this reckoning commences at the equinoctial point Aries. The longitude is reckoned from the same point along the ecliptic, and the latitude on great circles passing through its poles.

Let P be the pole of the equinoctial $r\ a$, and P′ the pole of the ecliptic $r'\ r\ b$; r the first point of Aries where they intersect; and S a star. Then $r\ a$ is called the right ascension, and a S the declination of that star; $r\ b$, b S, the longitude and latitude of the same respectively. If these planes were to intersect at r' the effect would be, that the longitudes of the stars, which are always estimated from the intersection of the planes of the equinox and ecliptic, or from the first point of Aries, must continually increase; and by comparing the longitudes of some of the stars at different times, the mean motion of the equinoctial points, or the precession of the equinoxes, may be discovered.

M. Lalande, in his Astronomy, has computed the precession by comparing the longitude of Spica Virginis as assigned by Hipparchus, with the longitude of the same star computed in 1750.

		s	°	
128 A. C.	Longitude of Spica Virginis	5	24	0
1750 A. D.	Ditto	6	20	21
	Increase in 1878 years . .	0	26	21

From this it appears that the annual mean precession is equal to $50''{\cdot}5 = \frac{26^\circ\ 21'}{1878}$. By a number of like comparisons, the same author fixed the secular precession—that is, the amount of the accumulated precession for 100 years, to be $1^\circ\ 23'\ 34''$; the mean annual precession corresponding to this is $50''\ 34$; and the sum of such annual precessions amounts to 1° in $71\frac{1}{2}$ years. If we suppose the precession to be $50''{\cdot}1$, then, in 25,869 years, $\left(= \frac{360 \times 60 \times 60 \times 10}{501}\right)$ the first point of Aries will have retrograded through an entire circle. The quantity $50''{\cdot}1$, which is the mean value of the precession, is obtained from the differences of the longitudes of a great many stars (three or four hundred, for instance) computed at different epochs. This mean quantity may not agree with the mean quantity derived from the observations of a single star, however many, or accurately made these observations may be. It will be found the case with Pollux, the second star in the following table. The differences, however, between the mean quantities of the precession as they result from three hundred stars or from a single one, is in all cases very small.



LONGITUDES.

STARS.	1815.				1756.				Difference of Longitude in 59 years.		Mean Annual Increase.
	s	°	′	″	s	°	′	″	′	″	″
α Arietis . . .	1	5	4	41	1	4	15	3	49	38	50·47
Pollux	3	20	39	48	3	19	50	55	48	53	49·70
Spica Virginis .	6	21	15	31	6	20	26	20	49	11	50·10
α Aquilæ . . .	9	29	9	53·8	9	28	20	6	49	47·8	50·80
α Pegasi . . .	11	20	54	31	11	20	5	19	49	12	50·10

Yet the difference which is found to exist, points out some peculiarity in every star. For instance, Pollux cannot be like most of the other stars, apparently entirely fixed, but must have what is called, or what we are obliged to call, from default of a knowledge of its cause, *a proper motion.* However, the comparison of the longitudes of the stars computed to the epochs of 1756 and 1815, establishes, as we have before observed, the important fact of the precession of the equinoxes. Because the mean longitude of a star is not altered solely by the precession of the equinoxes, astronomers employ the term *annual variation*, comprehending under it the effects both of *precession* and of *annual proper motion.*

We shall now compare the latitudes of the stars mentioned in the above table, at the same epochs.

LATITUDES.

STARS.	1815.				1756.			Difference for 59 years.
	°	′	″		°	′	″	″
α Arietis . . .	9	57	37·4	N.	9	57	3·2	+ 5·4
Pollux	6	40	18·4	N.	6	40	3	+ 15·4
Spica Virginis .	2	2	24·8	N.	2	2	6	+ 18·8
α Aquilæ . . .	29	18	35·5	N.	29	18	44	— 8·5
α Pegasi . . .	19	24	44	N.	19	25	44	— 4·0

It appears from this table that the changes of the latitudes are very small, in no case amounting to 0·4″ annually. "The astronomical fact is," says Woodhouse, "a minute annual change of latitude, and a considerable change of longitude. With regard to the former change we may conjecture, that it arises either partly from the precession of the equinoxes, and partly from other causes, or that it is altogether independent of the precession."

The succeeding tables show the *variation* both in right ascension and declination of the stars whose latitudes and longitudes we have just compared; these variations more clearly point out the general apparent change produced in the heavens by *the right motion of the earth's axis*, than those of the latitudes and longitudes, as the declination and right ascensions of the stars are reckoned in a manner similar to the latitudes and longitudes of places on the earth. Yet they are not in complete accordance with the *right motion*, as the latitudes of places have been supposed to remain fixed and from the dissatisfactory theory of corrections, of which we shall speak hereafter.

RIGHT ASCENSIONS.

STARS.	1843.	1817.	Difference in 26 years.	Mean Annual Variation.
	H. M. S.	H. M. S.	S.	S.
α Arietis . .	1 58 20·06	1 56 52·67	87·39	+ 3·36
Pollux . . .	7 35 42·09	7 34 6·06	96·03	+ 3·69
Spica Virginis	13 16 55·80	13 15 53·79	62·01	+ 2·38
α Aquilæ . . .	19 43 7·34	19 41 51·05	76·29	+ 2·93
α Pegasi . . .	22 56 56·64	22 55 38·95	77·69	+ 2·98

DECLINATION.

STARS.	1843.	1817.	Difference in 26 Years.	Mean Annual Variation.
	° ′ ″	° ′ ″	′ ″	
α Arietis . .	N. 22 43 4·14	N. 22 35 33·1	7 31·04	+ 17·34
Pollux . . .	N. 28 23 58·81	N. 28 27 31·7	3 32·89	— 8·188
Spica Virg.	S. 10 20 23·55	S. 10 12 6·5	8 17·05	+ 19·117
α Aquilæ .	N. 8 27 26·37	N. 8 23 37·6	3 48·77	+ 8·779
α Pegasi . .	N. 14 21 41·97	N. 14 13 26·0	8 15·97	+ 19·075

OF THE COLLAPSING OF THE PLANES OF THE EQUATOR AND ECLIPTIC.

The angle contained between the plane of the equator and ecliptic is what is denominated the obliquity of the ecliptic; which is shown, from repeated observations, to be variable. In this place it will be sufficient to show the results of a long succession of such observations by different astronomers, taken from the Encyclopedia Metropolitana.

Eratosthenes,	230	years	B.C.,	made	the	obli-	°	′	″
quity to be	-	-	-	-	-	-	23	51	20
Hipparchus,	-	B.C.	140	-	-	-	23	51	20
Ptolemy,	-	A.D.	140	-	-	-	23	51	10
Pappus,	-	—	390	-	-	-	23	30	0
Albatenius,	-	—	880	-	-	-	23	35	40
Arazachel,	-	—	1070	-	-	-	23	34	0
Prophatius,	-	—	1300	-	-	-	23	32	0
Regiomontanus,	-	—	1460	-	-	-	23	30	0
Waltherus,	-	—	1490	-	-	-	23	29	47
Copernicus,	-	—	1500	-	-	-	23	28	24
Tycho,	-	-	—	1587	-	-	-	23	29	30
Cassini (the father),			—	1656	-	-	-	23	29	2
Cassini (the son),			—	1672	-	-	-	23	28	54
Flamstead,	-	—	1690	-	-	-	23	28	48
De la Caille,	-	—	1750	-	-	-	23	28	19
Bradley,	-	-	—	1750	-	-	-	23	28	18
De la Lande,	-	—	1768	-	-	-	23	28	0
Pond,	-	-	—	1816	-	-	-	23	27	50
"Oliver Byrne,	-	—	1843	-	-	-	23	27	34"

The observations of Albatenius and Arazachel are here corrected for refraction. Those of Waltherus, and De la Caille, Lalande, computed. The obliquity of Tycho is put down as correctly computed from his observations ; also the obliquity as determined by Flamstead, is corrected for the nutation of the earth's axis ; these corrections Lalande applied. It is manifest, from the above observations, that the obliquity of the ecliptic continually decreases ; and "the irregularity which here appears," says the writer, "in the diminution, we may ascribe to the inaccuracy of the ancient observations, as we know they are subject to greater errors than the irregularity of this variation." If we compare the first and last observations, they give a diminution of 70″ in 100 years. If we compare the observations of Lalande with that of Tycho, it gives 45″. The same compared with Flamstead gives 50″. If we compare that of Dr. Maskelyne with Dr. Bradley's and Meyer's, it gives 50″. The comparison of Dr. Maskelyne's with that of Lalande, which he took as the mean of several results, gives 50″ as determined from the most accurate observations. The observations of Pond, compared with those of Bradley, give 44″, for the variation of the obliquity in 100 years, or 0·44″ annually.

Of Solar and Lunar Nutation.

(See Woodhouse's Astronomy, page 353, chap. XIV.)

The two inequalities that give title to the present subject,

are immediately, or rather intimately, connected with that of the preceding (on the precession of the equinoxes). Woodhouse says, "For the purpose of pointing out the connexion, we must look to the physical causes of these inequalities; and, in the inequable action of the cause of precession, we shall be able to trace the cause of solar and lunar nutations." "The actions of the sun and moon on the excess of the earth," which Woodhouse assumes to be "an oblate spheroid, above the" greatest "inscribed sphere, produce the retrogradation of the equinoctial points, or, as it is technically called, the precession of the equinoxes. The material circumstance in the production of these phenomenon is, that excess of the matter just spoken of. The other circumstances, scarcely less material, and, indeed, essential to the phenomenon, are the inclination of the sun's orbit to the equator, and the inclination of the moon's orbit to that of the sun's, and, consequently, to the earth's equator. If the sun and moon were constantly in the plane of the equator, there would, notwithstanding the earth's spheroidical form, be no precession. When either luminary is on the equator, its action in producing precession is nothing. Twice a-year, therefore—namely, at the two equinoxes, the sun's force in causing precession is nothing; and twice a-year, at the solstices, it is the greatest. It must, therefore, be of some mean value in the intermediate times. The retrogradation, therefore, of the equinoctial points, inasmuch as it arises from the sun, cannot be equable, since the cause producing it is, on no two successive days of the year, exactly the same. There arises, therefore, an inequality of precession. In consequence of such inequality, the precession in right ascension of α Arietis (taking one of the instances mentioned in Woodhouse's 13th chapter, page 352) on May the 20th, will not bear that proportion to the annual precession (3·34″) which the number of days between January 1, and May 20, bears to 365 days; and generally the precession for 50 days, whether it be in right ascension or in north polar distance, will not be necessarily equal to $\frac{50}{365} \times p$, p representing the precession. The exact portion of the annual precession (in right ascension or north polar distance) to which it is equal, or the correction necessary to be made to the mean portion, will depend on the season of the year to which the 50 days belong. The precession, therefore, after being used as a correction, itself requires to be corrected. This, however, is easily effected by altering the number by which (see page 349, Woodhouse's Astronomy) it is necessary to multiply the annual precession, in order to obtain its proportional part. Thus, of the star Serpentis, the annual precession in right ascension of which is 2·935″, the mean proportional precession on April 30, would be $\frac{120}{365} \times 2{\cdot}935 = {\cdot}328 \times 2{\cdot}935$, and 328 would be the multiplier:

but this is too large, the actual precession generated from January the 1st to April the 30th being less than the proportional part of the mean. It may be made duly less then, by merely lessening the multiplier ·328 : in the present instance, it would be reduced to ·300, which number and like numbers, in like instances, are furnished by proper tables (see Woollaston's Faciculus, Appendix, page 42). This, however, it is to be noted, is not the sole method for correcting the precession. The inequable retrogradation of the equinoctial points, or the inequality of the precession, is not the sole effect produced by the unequal action of the sun on the earth's excess of matter above its" greatest "inscribed sphere. The obliquity of the ecliptic, which, were the precession uniform, would not be effected by the cause producing precession, is subject to a semi-annual equation : since, as in the inequality of precession, the force causing a change in the obliquity arrives twice in a year at its maximum. These two effects, one an inequality of precession, the other an oscillation of the plane of the equator, constitute what is called the solar nutation."

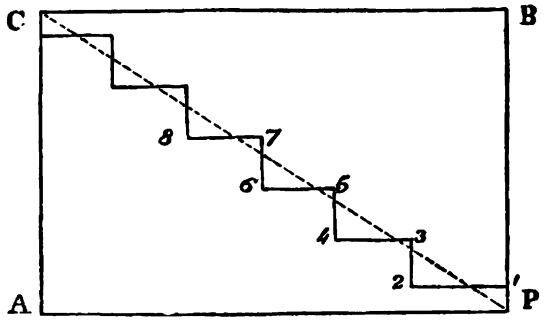
"There is also, as it may be conjectured from the arguments just alleged, a lunar nutation. The precession of the equinoxes is produced by the joint action of the sun and moon. As the sun not being in the equator, causes that part of the precession, which is due to his action, to be inequably generated ; so the moon, continually altering her declination, is continually causing precession with an unequal force. But the period of the inequality of its action, from an evanescent state to a state of maximum, is different from the period of inequality of the sun's action. It is no semi-annual period. The lunar period depends, however, on principles the same as those that regulate the solar. When the moon's orbit, which is continually changing its position, returns, at the end of any interval, to the same position which it had at the beginning, the interval so circumstanced is the period required. Now, this is regulated by the motion of the moon's nodes. The moon's orbit is inclined to the ecliptic, and its nodes retrograde, in about 18 years and 7 months. At the beginning, suppose the moon's node to have been in the node of the equator and ecliptic, then, at the end of 18 years and 7 months, the same node will have described 360 degrees contrary to the order of the signs, and returned to the first point of Aries ; and during this retrogradation of the node, the lunar orbit will have occupied every position which it can occupy relative to the equator. The inequality of the moon's action, then, in causing precession, will have passed through all its vicissitudes. But, as in the former case, this is not the sole effect of the inequality of the moon's force. The plane of the equator will be made to

oscillate: so that, according to the longitude of the node of the moon's orbit, it will be necessary to correct the mean obliquity on account of lunar nutation." Woodhouse continues to say, in reference to another part of his book, "We have seen in pp. 192, 193, that the phenomena of the precession can be accounted for, by supposing the pole of the equator to describe uniformly a small circle round the pole of the ecliptic in a period of 25,869 years. But these new phenomena of precession render some modifications necessary in the preceding hypothesis. By reason of the solar nutation, the pole of the equator will oscillate, during half a-year, about its mean place in the above-mentioned small circle, and the retrogradation of the pole will not be uniform. There will be a like oscillation, and a like inequality of precession from the lunar nutation, but for a longer period. From both causes, then, the north polar distance, and the right ascension of the stars, will be changed. In order to make the former the true precession, we must correct them both for solar and lunar nutation."

"We have, in the preceding pages, described the cause of solar and lunar nutations. But lunar nutation, which is by far the most considerable, was not found out from a previous persuasion or belief of the existence of its cause. Bradley, soon after the discovery of aberration of light, noticed it as a phenomenon, and then assigned its cause, and the laws of its variation. But the solar nutation has never appeared to astronomers as a phenomenon. It could scarcely be expected to be noticed as such, since its maximum is less than half a second. Its existence and quantity are derived from physical astronomy; and on such authority, it is introduced as a correction of astronomical observations." Woodhouse concludes this account by saying, "It has been proved, in confirmation of Bradley's conjectures, that the phenomena of nutation are explicable on the hypothesis of the pole of the earth describing round its mean place (that place which it would hold in the small circle described round the pole of the ecliptic, were there no inequality of precession) an ellipse, in a period equal to the revolution of the moon's nodes. The major axis of the ellipse is situated in the solstitial colure and equal to 19″·296: it bears that proportion to the minor axis (such are the results of theory) which the cosine of the obliquity bears to the cosine of twice the obliquity: consequently the minor axis will be 14″·364. These are M. Zach's numbers. Bradley's are 18″, 16.″ Maskelyne's, 19″·10. Laplace's, 19″·16. (See Mecanique Celeste, lib. v., page 351.)"

Now, *the right motion or change of the earth's axis*, is effected by the combined actions of the sun and moon on the excess of the earth over its greatest inscribed sphere, which excess will

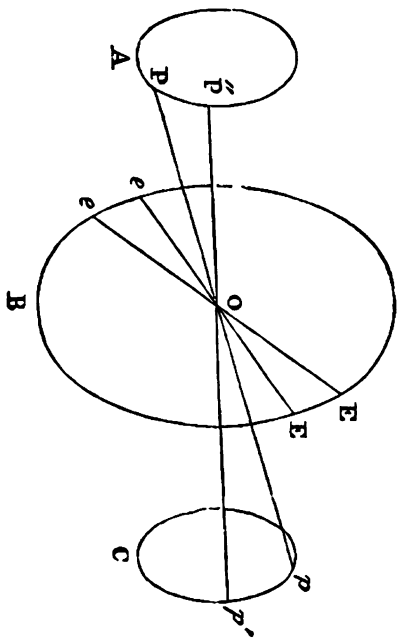
be shown hereafter to be in a continual state of change. Former theorists ascribe to this influence of the sun and moon, upon the excess above mentioned, the effects which we have just summed up from Woodhouse's Astronomy and the Encyclopedia Metropolitana; namely, "The precession of the equinoxes," "Solar and lunar nutation," and "The collapsing of the planes of the equator and ecliptic, or what is called the decrease of the obliquity of the ecliptic." Here there is but one effect ascribed to this combined action of the sun and moon. The following illustration will show, in a very simple manner, how different effects may appear to be produced by investigating causes separately that act jointly.



Let a material point P, be acted upon by two uniform forces, at the same time, in the directions P A, P B, and let the lines P A, P B, represent the magnitudes of these forces. It is well known, by what is technically called the parallelogram of forces, that it will not obey either of these forces, but the combined effect of both, and describe the diagonal P C. Now, the point P would arrive at C in precisely the same time, if the causes of action be investigated separately, even if at every alternate instant the forces alternately act; yet the path described by P would be very different from the true one, which the joint actions of the two forces cause it to describe. The zigzag path P, 1, 2, 3, &c., will be at best but an approximate to the true one P C.

This illustration of the combined action, viewed jointly and separately, simple as it is, compared with the joint actions of the sun, moon, and the other bodies in the solar system, on the excess before mentioned, shows how astronomers, by continually correcting a false hypothesis, have distorted the true motion, which we shall continue to explain. In stating that the earth's axis changes its position, we do not consider the mass moveable with it; no, the change of the axis, which is to be understood by what is termed its *right motion*, changes the position of the

equator, and, therefore, the latitudes of all places. It may be necessary to remark here, that the *right motion of the earth's axis* is very slow, so much so, that scarcely any perceptible difference in the latitudes can be observed in 100 years; chiefly on account of this motion not being recognised in a proper manner, which has tended to baffle the theory of corrections now so dissatisfactory. For the purpose of exemplification, we shall show how this motion of the earth's axis would cause the phenomena of a precession of the equinoxes. This may be shown by simply allowing the poles to change their position in two opposite circles. It may be necessary also to remark, that this supposition is made merely for the purpose of illustration, for the right motion of the earth's axis is not in a circle, but in a looped spiral curve of double curvature, the nature and properties of which are given in the Author's new work on the Calculus, before referred to. However, when this supposed plain motion is understood, *the right motion of the earth's axis* (which at once effects those apparent motions recognised by the terms, precession, nutation, and the decrease of the obliquity of the ecliptic,) can readily be conceived.



If a right line, P p, be supposed to revolve about its middle point O, in such a manner as to describe a circle A P P″; the other point p, will describe an equal circle, whose plane will be parallel to that of the former circle. The lines P p, P″ p', &c., will have the same inclination to the plane passing through O, parallel to the planes of these circles. Now, if a plane be supposed to pass through O, perpendicular to P p, and intersect the plane E B e, in E e; when the point P, moves to P″, the plane perpendicular to P″ p', passing through O, will intersect the plane E′ E B in E′ e': so that, if P p, P″ p', &c., be considered consecutive positions of the earth's axis, the plane of the ecliptic to be E B e, and the variable plane passing through O, perpendicular to P p, P″ p', &c., the plane of the equator, then the consecutive points E E′, &c., show the precession of the points where the planes of the

equator and ecliptic intersect, and how their positions are changed with the motion of the poles P *p*. A motion similar to this, but of a more compound nature, causes what is technically termed the precession of the equinoxes. Now, the spiral motion, or rather change of the earth's axis, above alluded to, which is chiefly caused by the influence of the sun and moon on the excess of the earth above its greatest inscribed sphere, is what we have designated *The Right Motion* or *Change of the Earth's Axis*. The nature of this motion can be readily conceived from the apparent phenomena of precession, nutation, &c. Hence, this exposition shows that the earth has but three motions ; namely, one round the sun, one on its axis, and a change of that axis : the nature of this last motion or change is something similar to that which is observed in a globular body spinning on an horizontal plane. However, the change of the axis in this globular body is only calculated to give an idea of the uniform change of the axis of the earth, caused, as we have before observed, by the constant and commanding influence of the sun and moon. As the poles shift their positions on the surface, so does the equator and the protuberance there ; and, in fact, every particle of which the globe, if we may so term it, is composed, endeavours to accommodate itself to this motion : so that the earth's true form is that which may not be improperly termed an exvoluted spheroid, continually, but slowly, changing its relative position. To guard against any misconception, it may be necessary to state, that the change of the protuberance at the equator, or rather the change of the excess of the earth above its greatest inscribed sphere, is not a sliding of that excess over the inscribed sphere, but the establishing of another and another excess in different positions, in consequence of the change of the position of the earth's axis, which is continually being changed by the influence of the sun and moon on the excess existing at any time. Every particle, internal and external, natural and artificial, is influenced by this motion : the sturdy edifices of man soon moulder down, or their foundations are depressed or elevated : the true cause is never assigned, when no local cause can be trumped up ; all is left upon what is commonly termed the ravages of old time, which lets nothing alone.

The changeable protuberance or excess, which is said to be about thirty-four miles more in diameter at the equator, than the diameter through the poles—this slow and constant changing of the excess of the earth above its greatest inscribed sphere, has done more in baffling the observations of man, and all his inquiries respecting the nature and figure of the earth, than any other circumstance which might be named. If the earth were composed of one uniform substance, ready to accommodate

itself to the behaviour of this change of protuberance, the true motion of the earth's axis would have been discovered long ago. The heterogeneous substances of which this planet is composed, and which are more and more changed with respect to colour, pliability, and form, with their varied positions by the influence of heat and cold in the great laboratory of nature, must in a greater or less degree, in every instance, point out the existence of this unerring law.

The change of this protuberance, or rather the change of all the particles with respect to the imaginary line about which this planet makes its daily revolutions, although very slow and gradual, effects a change in the whole heterogeneous mass, every instant, more or less, according to the pliability of the several parts: those parts which do not instantly change, from the rigidness of their nature, ultimately feel its sway; so that the earth would become, from what we have termed the right motion of its axis, a figure whose nature and proportions could be readily determined, were it composed of one yielding and uniform substance; but on account of its heterogeneousness, and the rigidity of some of its component parts compared with others, this figure, which we shall here designate an *exvoluted spheroid*, becomes in parts more or less slightly indented or elevated, according as these parts and those surrounding them are more or less sensitive of the constant disturbing cause.

Not only a change in the inanimate kingdoms is effected by the right motion of the earth's axis, but also a constant change in the animate: it is not asserting too much to say, that the gradual decline of nations is mainly to be attributed to the same cause; for this change, by its powerful influence, leaves places more or less suitable for the habitation of man and other animals. In the words of the justly celebrated Dr. Charles Hutton, "Amid all the revolutions of the globe, the economy of nature has been uniform, and her laws are the only things that have resisted the general movement. The rivers and the rocks, the seas and the continents, have been changed in all their parts; but the laws which direct those changes, and the rules to which they are subject, have remained invariably the same." The following facts and statements may tend to illustrate this theory. The gradual change of the fixed stars, especially that which is termed the north pole star (Ursa Minor), with respect to the apparent north and south, shows, that the axis of the earth is continually changing its position; this fact is not disputed, but the parallels of latitude on the surface of the earth are supposed to remain fixed, which is by this theory controverted.

In the "Boundehesch," a work containing the cosmogony of the Parses, and supposed to have been written by Zoroaster, it

is said that Ormusd formed the light between the heavens and the earth; that he made the sun, moon, and stars, and divided the latter (probably those near the ecliptic) into twelve constellations. Each star in the Zodiac is said to be seconded by 6,480,000 smaller stars, and all these are represented as soldiers ready to make war on the enemies of nature.

Ormusd, it is added, has also placed, in the four quarters of heaven, four sentinels to watch over the stars, — of these Taschter guards the east, Satevis the west, Venana the south, and Haftorang the north. There is said also to be a great star, Meschgâh, in the midst of heaven, for the purpose of giving further protection to the south when the enemy comes in great numbers. Now it is impossible to form an opinion what can be meant by this enemy so mysteriously announced; but the above designation of the stars seems to correspond with the *host of heaven* which is used in the Scriptures, and with the attendants or guards of the Supreme Deity, which is the *denomination* applied by the Egyptians to some of the constellations and planets; and it has been attempted by modern astronomers to prove, that four of the principal fixed stars were really situated in, or near, the four cardinal points of the horizon about the year 2200 B.C., which is the period assigned to the first Chaldean observations. Delambre remarks, that the longitude of Aldebaran, at that epoch, was 11° 20′, and its latitude 5° 30′ south; and as Antares differs from Aldebaran in longitude by six signs, and has 4° 30′ south latitude, it follows that these stars were, then, very nearly in the points of the vernal and autumnal equinoxes; consequently, one of them would be seen to rise near the east about the time that the other was setting a little to the north of the west. Now, it has been alleged that Taschter signifies the genius presiding over rain, and we know that the heliacal rising of Aldebaran was considered by the ancients as an indication of approaching storms; hence, it is with some propriety inferred that this star and Antares were two of those alluded to in the Persian story. The other two stars are less certain, *the right motion of the earth's axis not being recognised.* Delambre supposes these might be Fomalhaut and Regulus, which were then nearly in the plane of the solstitial colure, and the former would be visible in the south at an altitude of about twelve degrees above the horizon of Babylon, while Antares and Aldebaran were respectively rising and setting. But Regulus must have been 34° below the northern point of the horizon, supposing the axis not to change; consequently, according to this theory, Regulus would be visible at the same hour, in that latitude. If, therefore (continues Delambre), it was meant that the four stars were at once seen in the situations just mentioned, we

must look for some other star having the same longitude as Regulus, but having at least 34° of north latitude; the star γ in Ursa Major is so situated, and it is possible that this might be the star in question. M. Bailly observes, that the notion of the four stars guarding the heavens seems to have extended to China, for, in the "History of the Astronomy of the Celestial Empire," it is said, that there are four spirits which preside over the four seasons, meaning probably the quadrants of the Zodiac; and it is likely enough that this kind of observation would be made by *any* people among whom Astronomy was in its infancy.—*Narrien's Historical Account of Astronomy*, p. 138.

(Continued in the next number.)

A NEW THEORY OF THE STRENGTH AND STRESS OF MATERIALS.

This paper is taken from the "Civil Engineer and Architect's Journal," for June, 1846.

I DO not intend to occupy much space or time in dilating upon the importance of this subject, or in giving a history of its rise and progress, or in making apologies when I differ from my predecessors, however instructive, entertaining, or judicious an opposite procedure may be; but when I differ, I will give my reasons for doing so without circumlocutory apology.

Theory tells us that if a uniform bar—no matter what the figure of the cross section may be, or what substance it may be composed of—be suspended by one extremity, and loaded at the other till it is on the point of being torn asunder, the weight and the corresponding transverse sectional area are proportional. The bars or rods compared requiring only uniformity and equality of texture, we may lay down a general law, *the lateral resistances* (in the direction of a perpendicular to the transverse sections) *are in proportion to the areas of these transverse sections.* This law is very evident, for if a bar or rod were conceived to be longitudinally divided into any number of equal strips, no reason could be assigned why one of these strips should support a greater portion of the weight than any of the others, so that each would support an equal part of the weight, in the same manner as an assemblage of equal parallel ropes divide the weight of an appended body equally among them. Experiments on lateral strains prove these deductions to be correct, and it affords an instance in which theory and practice may be said to coincide. The contrary is the case when the beam or bar is supported in a horizontal position, for then, the

law of resistance, opposing fracture by an incumbent weight or force, is more difficult to establish, because we do not so readily see how the resisting forces exert themselves. Unlike lateral strains, the discrepancy existing between the results given by theory and by experiments is very great indeed ; so much so, that very little can be relied on the theoretical results that are beyond the range of experiments. Indeed, with experiments of a range sufficiently extensive, no very great mistake can be made, however loose and uncertain the theory may be ; but when we require a step far beyond our experiments, such as the determination of the best form and dimensions for a tubular bridge like that proposed by Mr. Stephenson,* then the want of a theory, supported by experiment, is a very great requirement.

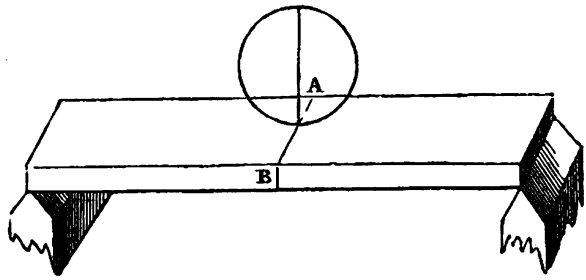


Fig. 1.

Galileo was of opinion that if a beam were supported at its extremities, as in fig. 1, and loaded by a weight at the middle, that all the fibres or filaments would exert equal resistances to prevent fracture, and that when these were overcome the whole would tend to turn about that boundary A B, in contact with the weight.

As this view of the subject supposes all the fibres to exert equal resistances, and in the direction of their lengths, these resistances will be so many equal and parallel forces and may be considered as acting together in the centre of gravity of the

* I regret to perceive, now success has attended the bold experiment of constructing tubular bridges, that attempts are being made to deprive Mr. Robert Stephenson of the merit so justly his due, as the original designer of the plan, and the superintender of its execution. Mr. Fairbairn, who, with Mr. Eaton Hodgkinson, assisted Mr. Stephenson in working out the design, claims to have the merit of all but the original idea. Now having paid much attention to Mr. Stephenson's experiment, and from conversations that I have had with him on the strength of materials, as a lover of fair play, I feel bound to say, from my own personal knowledge, that not only the original idea of tubular bridges, but also the mode of carrying it into effect, is due to him. O. B.

section, so that denoting the resistance of a single fibre by f, and supposing the section to be a rectangle of breadth b, and height h, then will $f\,b\,h$ express the sum of the resistances, and as this acts at the centre of gravity of the section, which is at the distance of $\frac{1}{2}h$ from the line A B, its moment to turn about A B will be $f\,b\,h \times \frac{h}{2} = \frac{f\,b\,h^2}{2}$. Leibnitz gave another hypothesis, which agreed with that of Galileo with respect to the position of the axis about which the segments would turn. But Leibnitz supposed the filaments or fibres to exert forces proportional to their distances from the axis ; so that the middle fibre, according to the theory of Leibnitz, exerted but half the force of the extreme fibre. Calling the force of the extreme fibre f, the sum of the forces would be $\frac{f\,b\,h}{2}$; and since the centre of such a system of parallel forces is at the distance of $\frac{2h}{3}$ from the axis about which the whole is supposed to turn, hence the moment to turn will be expressed by $\frac{f\,b\,h}{2} \times \frac{2h}{3} = \frac{f\,b\,h^2}{3}$. It is easily seen, that, as far as regards the comparative strength of rectangular beams of the same material, or of similar beams whose transverse sections are rectangular, it is no matter which of these hypotheses be adopted, for both point out the law of resistance to be as the breadth multiplied by the square of the height ; we shall in future use the term height or depth for the dimension in the direction of the pressure.

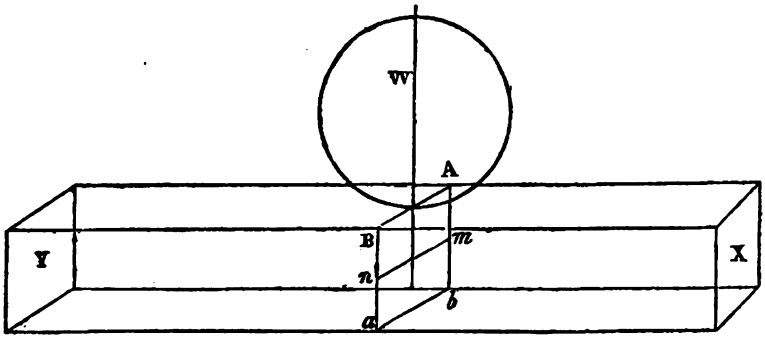


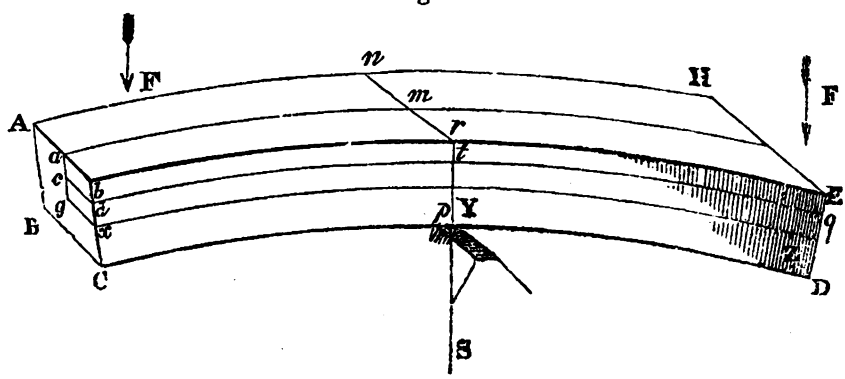
Fig. 2.

Galileo and Leibnitz supposed that the segments of a beam X Y, fractured by a weight W, turned about the line A B

where the fracture terminates. But James Bernoulli, Mariotte, and others, were of opinion that the segments had a tendency to turn about a line, as *n m*, entirely within the section ; the fibres on that side of the line where the fracture begins are extended, and those on the other side compressed. If the beam X Y, resting on two props at X and Y, be fractured by the weight W and *m n*, the line inside of the section A B *a b*, about which the segments of the beam have a tendency to turn, then the fibres or filaments in the space *n m b a* are supposed to be extended, and those in the space B A *m n* compressed. The imaginary line *m n*, which divides the section A B *a b* into two parts—the area of compression and the area of tension—is called the neutral axis. Mr. P. Barlow laboured much to find out the true position of this neutral axis in different sorts of timber. The result of his labours and experiments may be summed up in the following words, the truth of which is very questionable :—" The centre of tension and the centre of compression each coincide with the centre of gravity of its respective area (?) and the neutral line which divides the two is so situated, that the area of tension into the distance of its centre of gravity from the neutral axis, is to the area of compression into the distance of its centre of gravity from the same line, in a constant ratio for each distinct species of wood, but approximating in all towards the ratio of 3 to 1." (?) It would take up too much space to dwell on the absurd conclusions of Mr. Barlow ; there are one or two things which require but little consideration to detect : first, the so called neutral fibres do not arrange themselves in a right line in all forms of beams, indeed, if such a line did exist, it would be a curve, governed by the external form of the beam and the force applied. In the second place, the centre of gravity cannot agree in all cases with the centres of the forces of the filaments of the compressed and extended areas, and any man at first sight might suppose that the areas of compression and tension would bear a constant ratio to each other in each distinct species of wood. In the third place, I defy experiment either to confirm or contradict these conclusions of Mr. Barlow, for they have nothing whatever to do with the strength of beams. He merely says, Find where MY LINE is by experiment, and I will give you something like the result of experiment from a line so determined. Let *x* Y Z be a portion of a beam in the locality of fracture, caused by the forces F F acting in the directions of the arrows. The same process of reasoning which points out a neutral axis in the whole A H D C, will point out a neutral axis in any portion of the body *g a b x t q*, no matter where it be situated ; in fact, every fibre may be said to be compressed on one side and

extended at the other, while the whole or each is bent round a common centre, as S, entirely outside the body. Then S Y is the radius of curvature of the arc C *p* D at the point *p*.

Fig. 3.

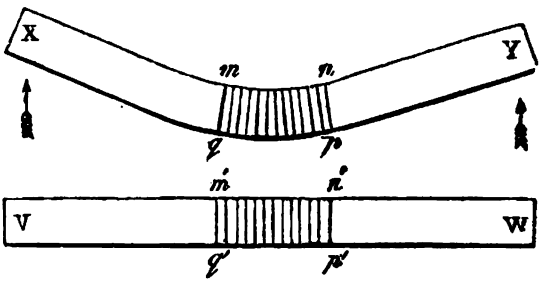


Now let us take *g c d x t Y q* Z, any portion of the beam, it is evident that the filaments in the upper part near to *d t q* are expanded, and those near to *x* Y Z are compressed; according to this reasoning there is a set of fibres between *d t q* and *x* Y Z which are neither compressed nor expanded; hence, each portion of the beam is entitled to a neutral axis, which is relatively correct, but each neutral axis is itself bent round a centre in *r* S.

It is stated by Tredgold ("Practical Essay on the Strength of Cast Iron," page 53),—"When a rectangular beam is supported at the ends, and loaded in any manner between the supports, it may be observed, that the side against which the force acts is always compressed, and that the opposite side is always extended; while at the middle of the depth there is a part which is neither extended nor compressed; or, in other words, it is not strained at all.

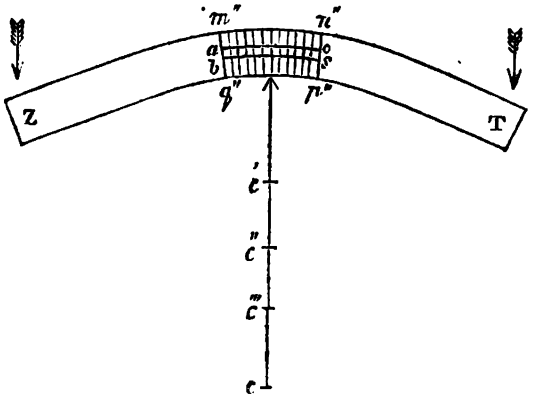
"Any one who chooses to make experiments may satisfy himself that this is a correct statement of the fact, in any material whatever, whether it be hard and brittle, as cast iron, zinc, or glass; or tough and ductile, as wrought iron and soft steel; or flexible, as wood and caoutchouc; or soft and ductile, as lead and tin. In very flexible bodies it may be observed by drawing fine parallel lines across the side of the bar before the force is applied; when the piece is strained, the lines become inclined, retaining their original distance apart only at the neutral axis." Now this fallacious statement (first made by M. Mariotte, which Mr. Barlow and a host of others down to Mr. Moseley have endeavoured to support by experiments, conjectures, and assertions,) may be exposed in the following descriptive manner, before giving the mathematical investigation. Suppose XY, VW, and ZT to be the same beam under

Fig. 4.



different circumstances. If a beam Z T be strained, with parallel lines drawn as above directed, at $m''n''p''q'$, the very fact, that the line $b\,s$ becomes bent, shows that its particles are not in a state of quiescence, and that it is under the influence of the force which bent it, although it may be the same length

Fig. 5.



as $m'n'$, or that one of the subdivisions of $b\,s$ is equal to one of those in $m'n'$, the position of the beam before it is bent. As Z T becomes more and more bent, the divisions will open in the upper regions near $m''n''$, and more contracted towards the lower near $q''p''$, so that the position of the neutral axis must shift towards the lower part of the beam from the top, unless the curves $m''n''$ and $p''q''$ be arcs of concentric circles, then the arc $a\,o$ in the centre will always be equal to $m'n'$. But it is not evident that the particles in the arc $a\,o$ or $b\,s$ are not strained at all, because either of them happen to be of the same length as $m\,n$, $m'n'$, or $m''n''$, which were all equal before the forces were applied.

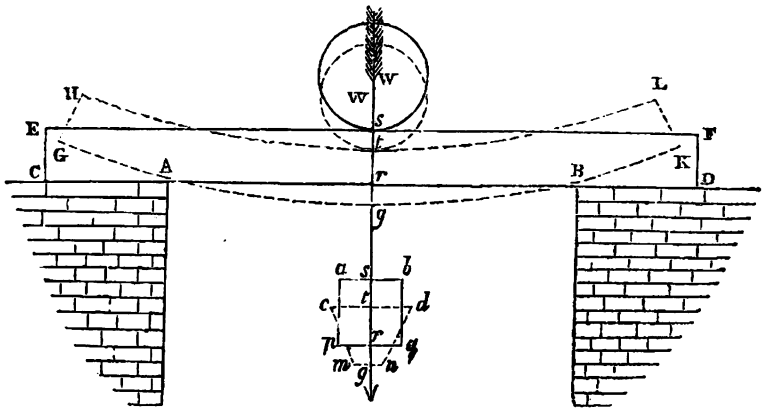
The conclusion we come to here is, that the whole turns round an axis, sometimes outside of the body and sometimes inside, according to the position of the centre of the circle of curvature of the curve where the greatest strain is applied. Let c' be the centre of the circle of curvature of the point in

the centre of the arc $q''p''$, then that arc is supposed to be strained round c' as an axis. Let c'', c''', and c be respectively the centres of the circles of curvature for the points in the middles of the arcs $b\,s$, $a\,o$, $m''n''$; then the arc $m''n''$ is supposed to be bent round the centre c; $a\,o$ round c''', and $b\,s$ round c''. When the filaments that are the most expanded,—that is, those near $m''n''$ in Z T, and near $q\,p$ in X Y,—become flatted at the middle, between m'', n'', or between q, p, which is generally the case before fracture ensues, then the centres of the circles of curvature at these centre points, after changing from c to c''' to c'', &c., as the beam Z T becomes more bent, now returns in a contrary direction, c', c'', c''', &c.; and when the fibres in $m''n''$ become straight, the radius of curvature becomes infinite, as in the case of the beam V W at rest, in which the particles or fibres at $m''n''$ are supposed to be straight.

We shall next consider the nature of the forces exerted by the filaments at different points of the cross section, in the region where fracture would ensue, when the strain exceeds the elastic limit of the body.

The beams, or bars, the nature of the cross sections of which we are investigating, are supposed to be supported at the ends and loaded in the middle. In small beams, the change in the particles that we are about to describe is not perceptible; yet it will be found very considerable in large girders, such as the tubular bridge about to be constructed by Mr. Stephenson, or in small girdles of a flexible nature.

Fig. 6.

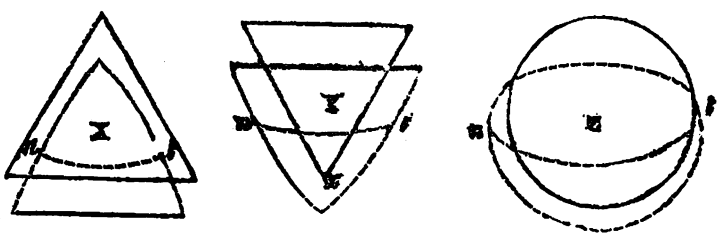


The figure used in elucidating this matter is distorted, in order that the change under consideration, near the centre of the beam, may be more apparent. A very simple mode of illustrating what we shall describe relative to the molecular action of the particles in a cross section, near the centre of the beam, may be obtained by taking a rectangular piece of

caoutchouc, india-rubber, whose cross section would be represented by *a b q p;* but, it is to be understood, that in point of structure we do not compare caoutchouc or india rubber with iron, brass, or wood, but merely to show the manner in which the particles in the cross sections of bodies, under the circumstances we have just described, endeavour to exert themselves. Let E C D F be the position of a beam before the weight W is applied, H G A B K L its position after the application; the cross sections in the two positions will be represented by the figures *a b q p* and *c d n m.* The action of the weight or force W compels the point *s* to move to *t*, and the point *r* to move to *g*, and has a tendency to lengthen the whole beam; while, at the same time, the filaments in the upper part of the beam, near the middle, become compressed in the direction of the length A B, and extended in the direction of the breadth *a b;* that is, the breadth *a b*, in one position, is represented by *c d* in the other. But the fibres in the lower part near *r*, in changing from *r* to *g*, become expanded in the direction of the length A B, and contracted in the direction of the breadth *p q*, so that *p q* in the cross section becomes *m n.* From the rigidity of materials, this change may not have place, or may not be perceptible; but, in all cases, a force acting in the direction of the arrow will have the tendency to change the cross section *a b q p* into one like *c d m n*, which if it be not able ultimately to effect, fracture must ensue.

As we have before observed, what we have just described will become clear by bending a rectangular piece of india-rubber. If the section of the rod or beam be circular, as Z, fig. 9, the change will differ materially from the one already described, for the circle will become, or endeavour to become, a figure like an oval. The change in the molecular particles in the cross section will arrange themselves differently, according to the external forms of the beams and the positions in which they are placed: something like the change of form in triangular beams are given in the succeeding figures 7 and 8. When the strain is about to exceed the limit of the

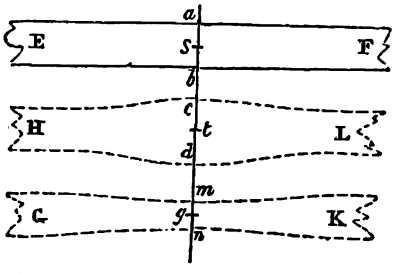
Fig. 7. Fig. 8. Fig. 9.



elastic power of the material, the fibres which are in a state of quiescence, compared with those in the extreme upper and

lower regions of the beam, will arrange themselves in a curve resembling *n t* in the figures 7, 8, and 9, the equation to which will be given hereafter. The behaviour of the force and filaments here stated are in themselves sufficiently simple and explicit; but, in order that none of our readers may enter upon the mathematical investigation of this important subject with incorrect notions respecting the different circumstances under which the beam may be placed, we have thought it expedient to add the following expositions and illustrations :—

Fig. 10.



Let E *s* F be the upper or lower surface of the beam before the weight W is applied; H *t* L will represent the upper, and G *g* K the lower, after its application. The same parts are marked with the same letters as figure 6 preceding, for these surfaces are supposed to have reference to that figure. If the particles at *a s b* in the bending process were such that they would merely become more dense, then the breadth at *a s b* would not be changed; but it is not the case, for the harder parts of the material merely obtrude themselves into the softer, and partly become compressed and partly swell the breadth of the beam near these parts, as at *c t d* in the upper section H *t* L. But in the lower surface G *g* K, or near it, the particles at *m g n* become separated, and the breadth becomes contracted from *a b*, which is equal to *p q*, fig. 6, to *m n*.

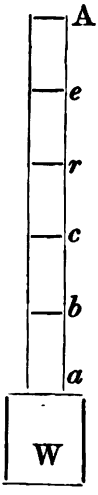
We shall in the next place proceed to the mathematical investigation. Let W be the weight in pounds that would be borne by a beam of wood, iron, or any other material, whose cross section is an inch square, when the strain, in the direction of its length, is as great as it will bear without destroying the elastic force of the body, and the direction of the force coincident with the length or axis of the bar; and let W′ be any other weight to be supported under the same circumstances. Suppose the cross section of the piece to support it to be a rectangle, whose breadth $= x$ and thickness $= y$ inches.

Then $W : W' :: 1 : xy$; or, $\frac{W'}{W} = xy$.

Strictly speaking, the lengths must be the same, or W and W′ must include the weights of their respective beams. This proportion has place from the well-known principle—abundantly proved by experiments—that "the strength of a bar or rod to resist a given strain, when drawn in the direction of its length, is directly proportional to the area of its cross section; while its elastic power remains perfect, and the direction of the force coincides with the axis."

Let n be the quantity of a bar of iron, or other material, an inch square, and a foot in length, represented by $a\,b$, would be extended by the force of the weight W in pounds, which, as in the last, is supposed to be the greatest, it would bear without destroying its elastic force; and let L be the length of any other portion of the same rod in feet.

Fig. 11.



Then $1 : L :: n : Ln$ = the extension of the rod, whose length is L. This is evident, for if we suppose the weight W to be attached at a, and to stretch $a\,b$ a quantity n it will stretch $a\,c$, $2\,n$; $a\,r$, $3\,n$, &c., because W, after stretching $a\,b$, will apply the same force to $b\,c$, and, strictly speaking, a little more, for it will have an additional weight in the length $a\,b$, a foot of the material; the same reasoning will hold with respect to any other foot, as e A for the weight W, and the weight of 4 feet of the material may be supposed to be applied at e, and act in expounding e A, but the weight of the rod is so small, that it may be neglected in moderate lengths. Strictly speaking, then, W, the ultimate force must involve the weight of the bar to which it is applied. The weight W would ultimately be reduced to zero, for the bar would only be able to support its own weight without destroying its elastic force.—Example: cast iron will bear without permanent alteration, 15,300 lb. upon a square inch; consequently the length of a rod of cast iron that would be just able to support itself without permanent alteration would be 4,896 feet, no matter what the cross sectional area may be; a cubic foot of cast iron weighs about 450 lb. Again, suppose W′ to be any other weight less than W,—

$W : W' :: n : \frac{W'n}{W}$ = the extension produced by the weight W′, supposing the length to be one foot, and the cross section an inch square; and also that the extension of a bar or rod by a force acting in the direction of its length, is proportional to the straining force; the area of the cross section remaining the same, and the strain not to exceed the elastic power of the beam, bar, or rod.

The extension for any length L, by the weight W′, $= \frac{LW'n}{W}$; for, $1 : L :: \frac{W'n}{W} : \frac{LW'n}{W}$.

W′ is supposed to involve the weight of the bar, as well as W.

Let it be required to find the elongation E, of a bar suspended vertically, and sustaining a given strain or weight W, in the direction of its length equal L feet, the influence of its own weight being taken into account, the sectional area A square inches. Without destroying the elasticity, or surpassing the

elastic limit, suppose ϵ to be the length that w pounds will elongate a bar weighing p pounds, one foot long and one square inch sectional area.

Suppose the last foot of a bar L feet long to be suspended to the second last, the effect of the last to elongate the second last will be found by the following proportion : $w : p :: \epsilon : \frac{p\epsilon}{w}$ = the elongation of the second foot.

Suppose these two feet to be attached to the third foot, we have $w : 2p :: \epsilon : \frac{2p\epsilon}{w}$ = elongation of the third foot; $w : 3p :: \epsilon : \frac{3p\epsilon}{w}$ = elongation of the fourth foot; therefore the sum of the following series will give the whole elongation, the length of the bar L representing the number of terms, we have $\frac{p\epsilon}{w}\left\{1+2+3+4\ldots.\mathrm{L}-1\right\} \times \frac{\mathrm{L}}{2} = \frac{\mathrm{L}^2 p\epsilon}{2w}$. Or the elongation may be determined thus :—Let x be the length in feet measured from the lower extremity, then $w : px :: \epsilon : \frac{\epsilon px}{w} =$ the elongation of a foot in length from the strain of the weight $p\,x$. And let $d\,x$, AS WRITERS ON THE CALCULUS SAY, BE THE LENGTH WHICH IS NEXT TO NOTHING,*

$$\therefore \underset{1}{\text{foot}} : \underset{dx}{\text{feet}} :: \frac{\epsilon px}{w} : \frac{\epsilon p}{w}\, x\,dx =$$

the elongation of the length which was supposed to be next to nothing. Integrating between the limits $x = o$ and $x = \mathrm{L}$; that is, $\int_o^{\mathrm{L}} \frac{\epsilon p}{w}\, x\,dx = \frac{1}{2}\,\frac{\epsilon p}{w}\,\mathrm{L}^2$. So that a bar 1 inch square and L feet long will be elongated by its own weight $\frac{p}{2w} \times \mathrm{L}^2\epsilon$; indeed, let the sectional area be what it may, the bar will be elongated by its own weight the same length, because the body is uniform, and each inch of sectional area is circumstanced in the same manner.

The number of pounds on each square inch of cross section $= \frac{\mathrm{W}}{\mathrm{A}}$, together with the weight of the bar up to that section. Then just at the point where the beam is suspended, there will be $\frac{\mathrm{W}}{\mathrm{A}} + p\,\mathrm{L}$ pounds on the square inch, which must not exceed w, the elastic limit in pounds for each square inch of section. $w : \frac{\mathrm{W}}{\mathrm{A}} :: \epsilon : \frac{\mathrm{W}\epsilon}{\mathrm{A}w}$ = the elongation of each foot in length by the

* On this point the reader is referred to O. Byrne's work on the "Calculus."

weight $\frac{W}{A}$ on the square inch; but as each square inch is strained by the same weight, each foot of the bar will be increased by the same length. $\therefore \frac{W\epsilon}{Aw}L =$ the elongation of the weight L, by the weight W. And hence, $\frac{W\epsilon}{Aw}L + \frac{p\epsilon}{2w}L^2 = L\frac{\epsilon}{w}\left\{\frac{W}{A} + \frac{p}{2}L\right\} = E$, the elongation by the weight W aided by $L \times A \times p$ the weight of the bar.

We shall next explain certain numbers introduced by writers on this subject, and called by them *moduli*. There is the *modulus* of elasticity, the *modulus* of resilience, the *modulus* of fragility, and the *modulus* of rupture.

The *modulus of elasticity* is the strain in pounds which would be required to extend a bar, one foot long and one square inch sectional area to double its length without altering its sectional area; or it is the trust in pounds that would compress the same unit bar into half its length, that is, till the foot becomes 6 inches, the sectional area remaining, as before, one square inch. We never could see the real use of introducing these imaginary numbers, unless for the purpose of mystifying the subject, or to make it assume a very scientific appearance.

It would be a stretch of the imagination to suppose a brick to be pulled till it would become the length of two bricks. He must be a very clever man indeed who determined the modulus of elasticity of pipeclay. Mere book-makers like Hall and Moseley, of King's College, cannot be offended; but men like Barlow and Hodgkinson, who have lost their time experimenting to find them, may be a little indignant to find their favourite numbers spoken of so lightly.

Let us suppose, for argument sake, that Tredgold is right with respect to the modulus of malleable iron; he found the modulus of elasticity to be 24,920,000 lb., or 7,550,000 feet of the same matter; sectional area one square inch. Those who have made experiments will give but little credit to one who finds him wrong. Tredgold found that 17,800 lb. on a square inch of good English iron would cause no permanent alteration, and would extend a foot or any other length, not taking into account the weight of the bar, the $\frac{1}{1400}$ part of its length. $\frac{1}{1400}$ in this case would be ϵ, $17800 = w$, and $p = \frac{475}{1444}$, for a cubit foot of malleable iron weighs 475 lb. nearly. Now it is evident, if the elastic limit of malleable iron were such as to allow it, and that the elongation was in direct proportion to its strain, that it would require 1,400 times 17,800 lb. to extend this foot of iron till it becomes two feet long, or, which is the same thing, till it becomes a foot elongated. Now 1,400

times 17,800 = 24,920,000 lb., or, $17{,}800 \div \frac{1}{1400} = 24{,}920{,}000 = \frac{w}{\epsilon}$; this, to make the matter assume a learned appearance, or rather a more college appearance, we shall represent by M_e; the modulus of elasticity and its reciprocal $\frac{\epsilon}{w}$ by $\frac{I}{M_e}$; so that

$E = L\frac{\epsilon}{w}\left\{\frac{W}{A} + \frac{p}{2}L\right\}$ may be written $E = \frac{L}{M_e}\left\{\frac{W}{A} + \frac{p}{2}L\right\}$.

A bar of malleable iron, one square inch of sectional area and 7,550,000 feet—nearly 1,430 miles long = 24,920,000 lb.; from which circumstance 7,550,000 feet is called the modulus in feet; $24{,}920{,}000 \div \frac{475}{144} = 7{,}550{,}000$. That is, $\frac{w}{\epsilon} \div p = \frac{w}{p\epsilon} = M_e \div p$; in this modulus Hodgkinson differs 400 miles of iron from Tredgold, and Barlow about 250 miles of iron from Hodgkinson. So much for the modulus of elasticity.

The elongation of a bar suspended vertically, and sustaining a strain of W lb.—the influence of its own weight not being taken into account—was found to be $\frac{\epsilon}{w} \cdot \frac{WL}{A}$, which we shall call l:—

$\therefore l = \frac{WL}{M_e A}$ by substituting for $\frac{\epsilon}{w}$ its value $\frac{I}{M_e}$. Before we explain what is meant by the *modulus* of resilience or fragility, it is necessary to say what is meant by a UNIT OF WORK. A pound weight raised vertically one foot is a unit of work done. A spring that lifts 1000 pounds $\frac{1}{1000}$ part of a foot does a unit of work. It is evident, that to elongate a bar an additional length l, the weight necessary to keep it elongated needs not be applied from the commencement of the process of elongating; for half l would be produced by half the weight, and one-third of l by one-third the weight, and so on. It is evident that W would be too much, for the weight W would not be actually employed in the business of elongation in moving from o to l.

Let L be the length of the bar before the weight W is applied; A, the area of its cross section; as before, l = elongation consequent on W. And let x be any elongation between o and l. The weight necessary to keep L stretched to $L + x$ will be $\frac{x}{l} \times W$; this weight may be said to

be fully employed while it passes through dx at the very extremity of $L + x$. $\therefore \frac{W}{l} x\,dx$ expresses the work done; $\therefore \frac{W}{l} \int_0^l x\,dx = \frac{W l^2}{2 l} = \frac{W l}{2}$, the whole units of work.

We have shown that $l = \frac{WL}{M_e A}$ $\therefore M_e A\, l = W L$

$\therefore W = \frac{M_e A l}{L}$ $\therefore \frac{W l}{2} = \frac{M_e A l^2}{2 L}$ = the units of work.

Also, the work done, or the resistance overcome, expressed in units of work, $= M_e \frac{A l^2}{2 L} = \frac{1}{2} M_e \left(\frac{l}{L}\right)^2 L A$; hence it is evident that the amount of work to be done to elongate different bars of the same material, any fractional part of its length expressed by $\frac{l}{L}$, which must not exceed the elastic limit, will vary as $L \times A$; for in every case $\frac{1}{2} M_e \left(\frac{l}{L}\right)^2$ remains constant.

The work done in elongating a bar to its elastic limit, whose sectional area is one square inch, and length one foot, equal to $\frac{1}{2} M_e \left(\frac{l}{L}\right)^2$, supposing the limit to be expressed by the fraction $\frac{l}{L}$; in this case $A \times L = 1$. The units of work done in elongating any other bar of the same material as the last, whose length equal L, and sectional area equal A, to its elastic limit, $= \frac{1}{2} M_e \left(\frac{l}{L}\right)^2 AL$. It is evident that $\frac{l}{L}$ is the same value as in the last case, hence, $\frac{1}{2} M_e \left(\frac{l}{L}\right)^2$ is the same in all cases with bars of the same material, elongated to their elastic limit. $\frac{1}{2} M_e \left(\frac{l}{L}\right)^2$ is called the modulus of resistance, and may be written M_r, and is sometimes called the modulus of longitudinal resilience. If the work of elongating goes on till fracture is produced M_f, the work necessary to cause fracture in a bar a foot long and an inch square, is styled the modulus of fragility, so that $M_f L A$ will express the units of work that will cause fracture, in the same manner that $M_r L A$ expresses the units of work to extend a bar to its elastic limit. By-and-by we shall explain

what is understood by the term Modulus of Rupture in Transverse Strains. The modulus of fragility, M_f must not be mistaken for the unit of tenacity, which is the number of pounds that would tear asunder a bar one square inch in section. Length is not taken into account in the unit of tenacity, but as the greatest strain is supposed to be on the highest section of the bar, it is therefore at this section the rupture will take place, hence the length must be taken into account in determining the weight that produces fracture.

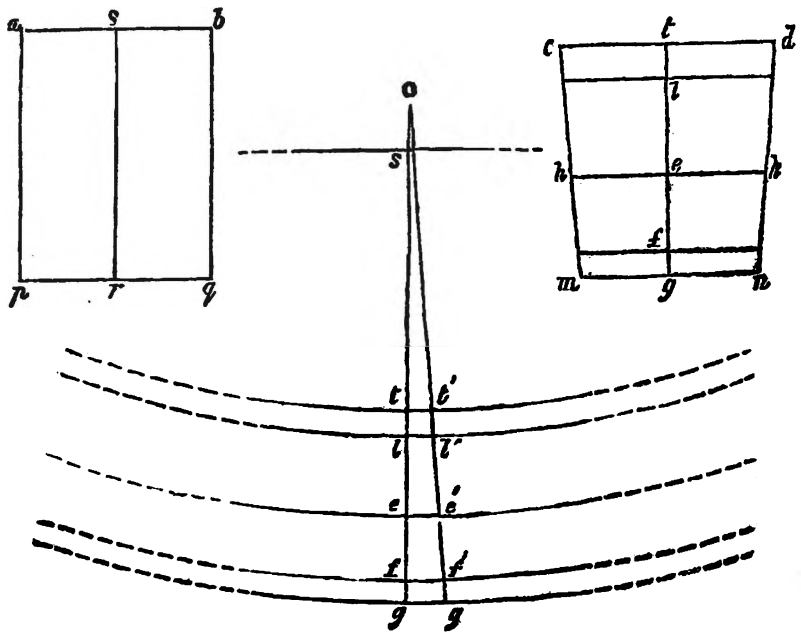
CONDITION OF RUPTURE IN A RECTANGULAR BEAM UNDER A TRANSVERSE STRAIN.

Let us next consider the nature of the forces exerted by the filaments of the cross section, near g, when the beam is in the state bordering upon rupture.

Let g' be any point of the curve $g\,g'$, very near g; and let the normals $t\,e\,g$ and $t'\,e'\,g'$ meet in the point o. Let ρ = the radius of curvature of the mean filament, $e\,e'$, at the point e, that is, the filament which is in a middle state between $t\,t'$ and $g\,g'$; it is represented by $h\,k$ in the cross section through $t\,g$; $h\,k$ is in such a position that it is equal in length to $a\,b$ or $p\,q$, the breadth of the beam before the weight W was applied.

It must be observed that fig. 12 is part of fig. 6 enlarged,

Fig. 12.



the same letters represent the same parts, and that $e\,e'$ is not considered neutral, but subject to peculiar vicissitudes before described, on account of the applied force. Let $\sigma = e\,e'$; $x =$ the distance of any filament whatever as $f\,f'$, from the mean filament, and σ' the length of $f\,f'$.

$o\,f' = \rho + x =$ radius of curvature of $f\,f'$, at the point f, then,

$$\sigma : \sigma' :: \rho : \rho + x$$

$$\therefore\ \sigma'\,\rho = \sigma\,\rho + x\,\sigma,\ \sigma' = \sigma + \frac{x\,\sigma}{\rho}.$$

On account of the forces applied, the longitudinal filaments undergo small dilations, which will destroy the equality that existed between σ and σ' before the forces were applied. Let $a =$ the primitive magnitude, then, supposing δ and δ' very small fractional parts of a, so that

$$\sigma = a\,(1 + \delta) \text{ and } \sigma' = a\,(1 + \delta')$$

$$\therefore\ a\,(1 + \delta') = a\,(1 + \delta) + x\frac{a\,(1 + \delta)}{\rho}$$

$$\therefore\ \delta' \quad \delta + \frac{x\,(1 + \delta)}{\rho} = \delta + \frac{x}{\rho} + \frac{x\,\delta}{\rho}.$$

Let $t\,t' = a\,(1 - \epsilon')$; $g\,g' = a\,(1 + \epsilon)$; $l\,l' = a\,(1 - \delta'')$; and let the height of the fibre at $g = i$, and breadth $a\,b$ or $p\,q = \beta$, that is, the breadth before the force is applied. $e\,f = x$; $e\,l = y$; $e\,g = a_1$; and $e\,t = a_2$. When the element or fibre at g, becomes expanded till its length a becomes $a\,(1 + \epsilon)$, then its breadth will be $\frac{\beta}{(1+\epsilon)^{\frac{1}{2}}}$, and height $\frac{i}{(1+\epsilon)^{\frac{1}{2}}}$, the height i is so small, this supposes the cross section of the extreme fibre, before the expansion takes place, to be similar to it after being expanded,—mind we say similar, not equal, for $\frac{\beta}{(1+\epsilon)^{\frac{1}{2}}}$ is less than β ; so the breadth is diminished. It will be perceived that we are here speaking of the lower filaments at g.

$$a_1 : (a_1 - x) :: \beta - \frac{\beta}{(1+\epsilon)^{\frac{1}{2}}} : \frac{a_1 - x}{a_1}\left(\beta - \frac{\beta}{(1+\epsilon)^{\frac{1}{2}}}\right)$$

$$\therefore \text{the breadth of the fibres at } f\,f' = \frac{\beta}{(1+\epsilon)^{\frac{1}{2}}} + \frac{a_1 - x}{a_1}\left(\beta - \frac{\beta}{(1+\epsilon)^{\frac{1}{2}}}\right)$$

$$= \frac{\beta}{a_1}\left\{a_1 - \left(1 - \frac{1}{(1+\epsilon)^{\frac{1}{2}}}\right)x\right\}.$$ The force arising from the action of $f\,f'$ is proportional to δ', and may be represented by $m\,\delta'$, m being a constant depending upon the nature of the material of which the bar consists ; therefore, the normal force which corresponds to the point f, normal to $t\,g$, is represented by

$\frac{\beta}{a_1}\left\{a_1-\left(1-\frac{1}{(1+\epsilon)^{\frac{1}{3}}}\right)x\right\}m\delta' dx$, and the moment of all the forces normal to eg with respect to the axis hk put $= M =$

$$\frac{m\beta}{a_1}\int_0^{a_1}\left\{a_1-\left(1-\frac{1}{(1+\epsilon)^{\frac{1}{3}}}\right)x\right\}\delta' x\,dx$$

but $\delta' = \delta + \frac{(1+\delta)x}{\rho}$; hence, by substitution,

$$M = \frac{m\beta}{a_1}\int_0^{a_1}\left\{a_1-\left(1-\frac{1}{(1+\epsilon)^{\frac{1}{3}}}\right)x\right\}x\left(\delta+\frac{(1+\delta)x}{\rho}\right)dx$$

Therefore M equal

$$\frac{m\beta}{12\rho}\left\{\left[6-4\left(1-\frac{1}{(1+\epsilon)^{\frac{1}{3}}}\right)\right]\delta\rho a_1^{\frac{3}{2}}+\left[4-3\left(1-\frac{1}{(1+\epsilon)^{\frac{1}{3}}}\right)\right](1+\delta)a_1^3\right\}$$

but $\rho : \rho + a_1 :: a(1+\delta) : a(1+\epsilon)$;

$$\therefore 1+\delta = \frac{\rho(1+\epsilon)}{\rho+a_1} \text{ and } \delta = \frac{\rho\epsilon - a_1}{\rho+a_1}$$

$$\therefore M = \frac{m\beta a_1^2}{12(\rho+a_1)(1+\epsilon)^{\frac{1}{3}}}\times$$

$$\left\{\left[2(1+\epsilon)^{\frac{1}{3}}+4\right](\rho\epsilon-a_1)+\left[(1+\epsilon)^{\frac{1}{2}}+3\right](1+\epsilon)a_1\right\}$$

This being established, we shall next determine the nature of the forces between e and t, or on the area $hk\,dc$. The breadth of the extreme fibres at $tt' = \frac{\beta}{(1-\epsilon_1)^{\frac{1}{3}}}$; $ot = \rho - a_2$; $ol = \rho - y$.

$$a_2 : y :: \frac{\beta}{(1-\epsilon_1)^{\frac{1}{3}}}-\beta : \frac{y}{a_2}\left(\frac{1}{(1-\epsilon_1)^{\frac{1}{3}}}-1\right)\beta$$

$$\therefore \frac{y}{a_2}\left(\frac{1}{(1-\epsilon_1)^{\frac{1}{3}}}-1\right)\beta+\beta = \frac{\beta}{a_2}\left\{a_2+\left(\frac{1}{(1-\epsilon_1)^{\frac{1}{3}}}-1\right)y\right\} =$$

the breadth of the fibres at ll'. The elastic forces of compression between e and t, as well as those of extension between e and g, tend to turn the surface of rupture in the same direction about the axis of rupture. The moment of all the forces normal to et, with respect to the axis hk, which we shall call M_1, is equal

$$\frac{m_1\beta}{a_2}\int_0^{a_2}\left\{a_2+\left(\frac{1}{(1-\epsilon_1)^{\frac{1}{3}}}-1\right)y\right\}\delta'' y\,dy;$$

$$\text{but } \delta'' = \frac{\epsilon_1 \rho - a_2 + (1-\epsilon_1)y}{\rho - a_2},$$

$$\text{for } \rho - a_2 : \rho - y :: a\,(1-\epsilon_1) : a\,(1-\delta'').$$

$$\therefore M_1 = \frac{m_1 \beta}{a_2(\rho - a_2)} \times$$

$$\int_0^{a_2} \left\{ a_2 + \left(\frac{1}{(1-\epsilon_1)^{\frac{1}{2}}} - 1 \right) y \right\} \left\{ (\epsilon_1 \rho - a_2) + (1-\epsilon_1) y \right\} y\, dy.$$

$$\therefore M_1 = \frac{m_1 \beta a_2^2}{1\,2\,(\rho - a_2)\,(1-\epsilon_1)^{\frac{1}{2}}} \times$$

$$\left\{ \left[2\,(1-\epsilon_1)^{\frac{1}{2}} + 4 \right] (\epsilon_1 \rho - a_2) + \left[+3 + (1-\epsilon_1)^{\frac{1}{2}} \right] (1-\epsilon_1) a_2 \right\}$$

Now the elastic forces M and M_1, when the beam is on the point of breaking, together with the pressure applied at B or A (fig. 6), which is equal to half the weight W and half the weight of the beam.

Representing, therefore, by w the weight of the beam,

$$M + M_1 = \tfrac{1}{2}\,(W + w) \times rB$$

Let $r\,B = p$ = half the distance between the supports = the perpendicular let fall from the axis round which the section of rupture turns upon the direction of the pressure at A or B. In the next number will be pointed out the erroneous principle upon which Hooke's law is founded.

(To be continued.)

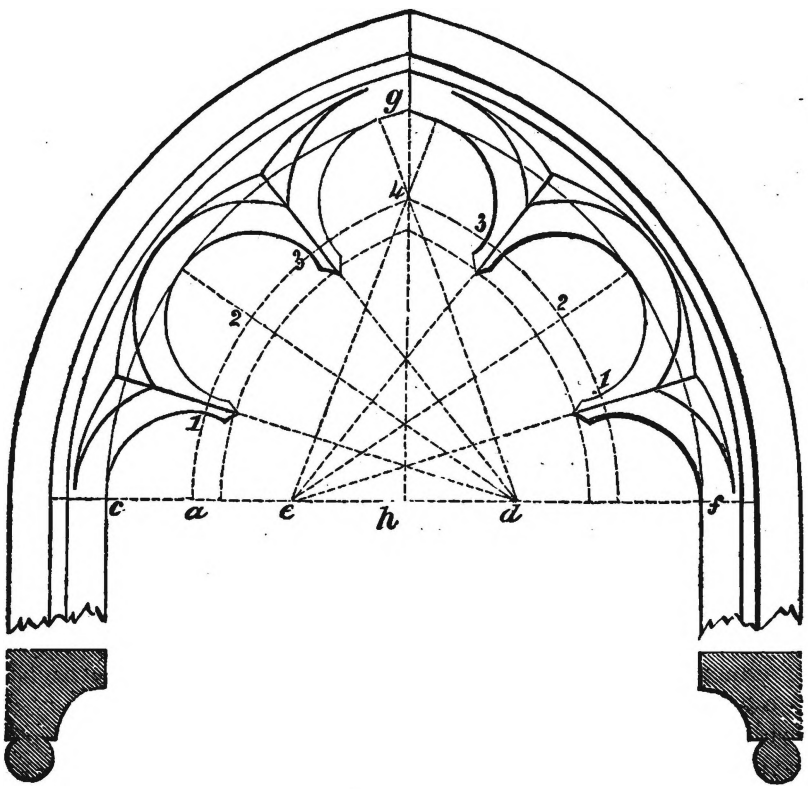
NOTE :—It is worthy of remark, that when the experiments were concluded respecting the Menai Tubular Bridge, a writer on that subject, in "The Civil Engineer," who at first opposed Mr. B.'s views, writes thus :—

"It was observed during one of the recent experiments that the top or compressed side of the tube bulged out transversely ; and it may be seen that with a bridge of the construction here suggested, the masses at the upper angles would, when in a state of compression, tend to be similarly bent. They would be likely to be bowed outwards or inwards, and this tendency is restrained by the horizontal bracing. These considerations confirm, to a certain extent, the views of Mr. Byrne, recently propounded in this Journal. It cannot be doubted that his general views respecting the horizontal transversal strains to which deflected beams are subject, when they tend to break rather by the distortion of the material than the disruption of it, form a valuable addition to the theory of the strength of beams."

Oct. 1846.

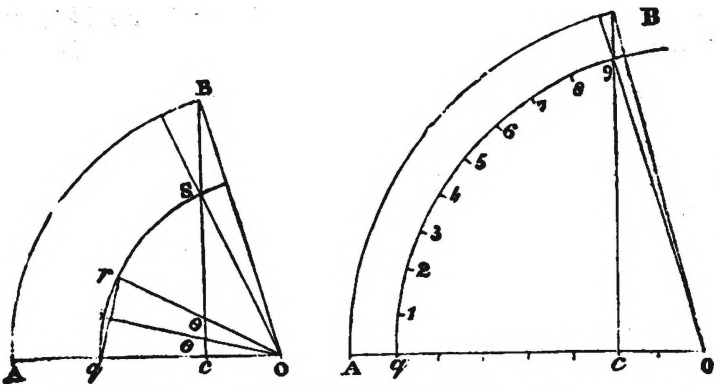
DIAGRAM FOR DRAWING GOTHIC ARCHES.

From the "Civil Engineer and Architect's Journal," September, 1844.



"To find the radius of an arc $a4$, concentric with a given arc $c\,g$, to meet the sine $g\,h$, so that the difference of the radii $c\,a$, may divide $a4$, the arc to be found into a given number of equal parts."

(We are indebted to our valuable correspondent O. T. for the above method of drawing a cinquefoil head; it is copied from a diagram made by Peter Nicholson, in whose handwriting is also the problem annexed. It will be observed that the diagram does not suggest any method of solving the problem; we therefore referred it to Mr. Oliver Byrne, who has favoured us with the following solution.)"



Let $A\,o = m$, $q\,o = x$, $c\,o = e$, and the angle $q\,o\,r = 2\,\theta$.

$\therefore A\,q = q\,r = 2\,x \sin\theta = m - x$; and $x = \dfrac{m}{1+2\sin\theta}$. Again $c\,S = \sqrt{x^2 - e^2} = x \sin 2\,n\,\theta$, ($n$ being the number of equal parts into which the arc $q\,S$ has to be divided);

$$x^2 - e^2 = x^2 \sin^2 2\,n\,\theta$$

$$x^2\,(1 - \sin^2 2\,n\,\theta) = e^2$$

$$\therefore x^2 = \frac{e^2}{1-\sin^2 2n\theta} = \frac{e^2}{\cos^2 2n\theta}$$

$$\therefore x = \frac{e}{\cos 2\,n\theta} = \frac{m}{1+2\sin\theta}$$

$$\text{And } \therefore \frac{m}{e} \cos 2\,n\,\theta - 2\sin\theta = 1. \qquad \text{(Q.)}$$

Although the general solution of the equation involves some difficulty, yet the value of θ or of $2n\theta$, and therefore of x, may be readily determined in all cases, to any required degree of exactness, by approximation; for, from the near approach of A o S to A o B, which is known, two values of θ may be readily selected, which, when substituted in (Q), will give results respectively less and greater than unity. One or two examples will show the certainty and simplicity of this mode of proceeding.

I. Let $m = 4$, $e = 1$, and $n = 9$, $\frac{1}{4} = \cdot 2500000 = \cos$ A o B $=$ 75° 31′ nearly. Equation (Q) becomes $4 \cos 18\,\theta - 2 \sin\theta = 1$

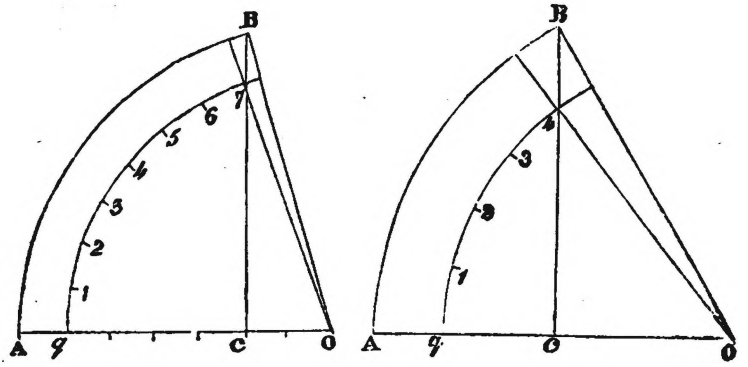
$\dfrac{75°\ 31'}{18} = 4°\ 11' = \theta$ nearly.

Let us substitute 4° 10′, and 4° for θ in (Q.) then we have

$$4 \cos 72 - 2 \sin 4°\ 0' = 1{\cdot}0965550\ (a)$$
$$4 \cos 18\theta - 2 \sin\theta = 1{\cdot}0000000\ (b)$$
$$4 \cos 75° - 2 \sin 4°\ 10' = \cdot 8899600\ (c)$$

$$75° - 72° = 3° = 180' \begin{cases} \cdot 2065950 = (a-c) \\ 0965550 = (a-b) \end{cases}$$

As 206595 : 96555 :: 180′ : 84′·125 = 1° 24′·125, which when added to 72° = 73° 24′·125 = 18 θ . ·. θ = 4° : 4′·673 and $x = \frac{e}{\cos 2n\theta} = \frac{1}{\cos 73°\ 24'\cdot 125} = 3\cdot 5008$ nearly, so that if A O be divided into 8 equal parts, A q one of them, will divide the arc q, 1, 2, 3, &c., into 9 equal parts.



II. Let $m = 7$, $e = 2$, and $n = 7$, $\frac{2}{7} = \cdot 2857142 = \cos$ A o B = cos 73° 24′ nearly. $\frac{73°\ 24'}{14} = 5°\ 14 = \theta$ nearly.

Let us substitute 5° and 5° 10′ for θ in equation (Q_2) which becomes $\frac{7}{2}\cos 14\theta - 2\sin\theta = 1$, and we have

$\frac{7}{2}\cos 70° : 0' - 2\sin 5°\ 0' = 1\cdot 0227589\ (a)$
$\frac{7}{2}\cos 14\theta - 2\sin\theta = 1\cdot 0000000\ (b)$
$\frac{7}{2}\cos 72°\ 20' - 2\sin 5°\ 10' = \cdot 8820694\ (c)$

$72° : 20' - 70° = 2°\ 20' = 140'$ { $\cdot 1406895 = (a) - (c)$; $\cdot 0227589 = (a) - (b)$

As 1406895 : 227589 :: 140′ : 22′·647.

·. 70° 22′·647 = 14 θ and θ = 5° : 1′ 617 hence $x = \frac{e}{\cos 2n\theta}$

$\frac{2}{\cos 14\theta} = \frac{2}{\cos 70°\ 22'\cdot 647} = 5\cdot 9572$ nearly = oq.

III. Let $m = 2$, $e = 1$, and $n = 4$. $\frac{1}{2} = \cdot 5000000 = \cos$ A o B = cos 60°. $\frac{60°}{8} = 7°\ 30' = \theta$ nearly.

If 6° and 7° be substituted for θ in equation (A_1) which in this example becomes $2\cos 8\theta - 2\sin\theta = 1$, we shall have

$2\cos 48° - 2\sin 6° = 1\cdot 1292042\ (a)$
$2\cos 8\theta - 2\sin\theta = 1\cdot 0000000\ (b)$
$2\cos 56° - 2\sin 7° = \cdot 8746472\ (c)$

$7° - 6° = 1° = 60'$ { $\cdot 2545570\ (a) - (c)$; $\cdot 1292042\ (a) - (b)$

2545570 : 1202042 :: 60′ : 31′ .·. $\theta = 6°31'$, and $8\theta = 52°8'$.

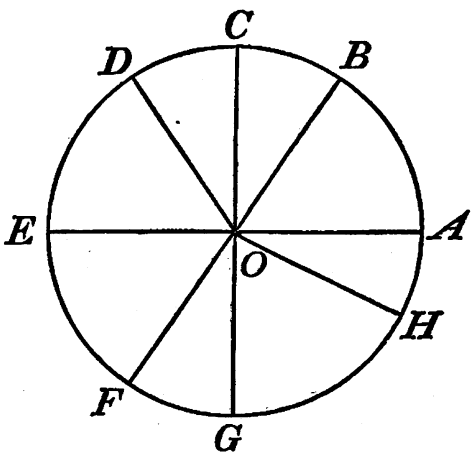
Since $q\,o = x = \frac{e}{\cos 8\theta} = \frac{1}{\cos 52°8'} = 1{\cdot}6291 = \frac{13}{8}$ nearly, this last example shows that great nicety is not required in selecting the angle q o 4, even when the number of parts are few and the angle A o B small; for in one instance we have supposed q o 4 to be 56° and in the other 48°, and yet we have arrived at the value of θ to a minute of a degree, which at once shows the certainty of this plan of proceeding. The geometrical construction of this problem becomes very easy by the use of the new proportional compasses. *(See page 79.)*

SPHERICAL TRIGONOMETRY TAUGHT IN ONE LESSON.

This Article is taken from a short Practical Treatise on Spherical Trigonometry. Published by J. A. VALPY. London : 1835.

As the sides and angles of spherical triangles are measured by circular arcs, and as these arcs are often greater than 90°, it may be necessary to mention one or two particulars respecting them.

The arc C B which when added to A B make up a quadrant or 90° is called the complement of the arc A B; every arc will have a complement, even those which are themselves greater than 90°, provided we consider the arcs measured in the direction A B C D, &c., as positive, and consequently those measured in the opposite direction as negative. The complement B C of the arc A B commences at B, where A B terminates, and may be considered as generated by the motion of B, the extremity of the radius O B, in the direction B C. But the complement of the arc A D or D C, commencing in like manner at the extremity D, must be generated by the motion of D in the opposite direction, and the angular magnitude A O D will here be diminished by the motion of O D, in generating the complement; therefore the complement of A O D or of A D may with propriety be considered negative.



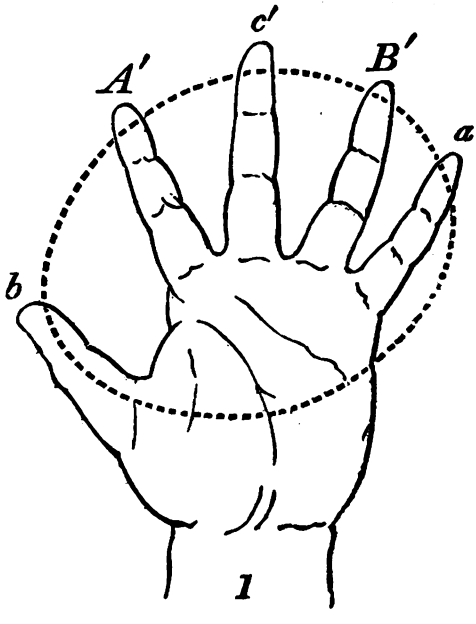
Calling the arc A B or A D, θ, the complement will be $90° - \theta$; the complement of 36° 44′ 33″ is 53° 15′ 27″; and the complement of 136° 27′ 39″ is negative 46° 27′ 39″.

The arc B E, which must be added to A B to make up a semicircle or 180°, is called the *supplement* of the arc A B. If the arc is greater than 180°, as the arc A D F its supplement, F E measured in the reverse direction is negative. The expression for the supplement of any arc θ is therefore $180° - \theta$; thus the supplement of 112° 29′ 35″ is 67° 30′ 25″, and the supplement of 205° 42′ is negative 25° 42′.

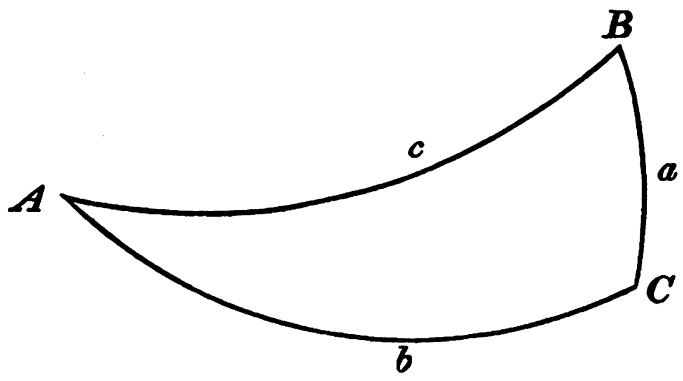
In the same manner as the complementary and supplementary arcs are considered as positive or negative, according to the direction in which they are measured, so are the arcs themselves positive or negative; thus, still taking A for the commencement, or *origin*, of the arcs, as A B is positive, A H will be negative. *In the doctrine of triangles* we consider only positive angles or arcs, and the magnitudes of these are comprised between $\theta = 0$ and $\theta = 180°$; but in the general theory of angular quantity, we consider both positive and negative angles, according as they are situated above or below the fixed line A O, from which they are measured, that is, according as the arcs by which they are estimated are positive or negative. Thus the angle B O A is positive, and the angle A O H negative. Moreover, in this more extended theory of angular magnitude, an angle may consist of any number of degrees whatever; thus, if the revolving line O B set out from the fixed line O A, and make n revolutions and a part, the angular magnitude generated is measured by n times 360°, plus the degrees in the additional part.

In a right-angled spherical triangle we are to recognise but five parts, namely, the three sides a, b, c, and the two angles A, B; so that the right angle C is omitted. *(See next page.)*

Let A′, c', B′, be the complements of A, c, B, respectively, and suppose b, a, B′, c', A′, to be placed on the hand, as in the annexed figure, and that the fingers stand in a circular order, the parts represented by the fingers thus placed are called Napier's circular parts.



If we take any one of these as a middle part, the two which lie next to it, one on each side, will be *adjacent* parts. The two parts immediately beyond the adjacent parts, one on each side, are called the opposite parts.



Thus, taking A′ for a middle part, *b* and *c*′ will be *adjacent* parts, and *a* and B′ are opposite parts.

If we take *c*′ as a middle part, A′ and B′ are adjacent parts, and *b*, *a*, opposite parts.

When B′ is a middle part, *c*′, *a*, become adjacent parts, and A′, *b*, opposite parts.

Again, if we take *a* as a middle part, then B′, *b*, will be adjacent parts, and *c*′, A′, opposite parts.

Lastly, taking *b* as a middle part, A′, *a*, are adjacent parts, and *c*′, B′, opposite parts.

This being understood, Napier's two rules may be expressed as follows:—

I.

Rad × sin middle part = product of tan adjacent parts.

II.

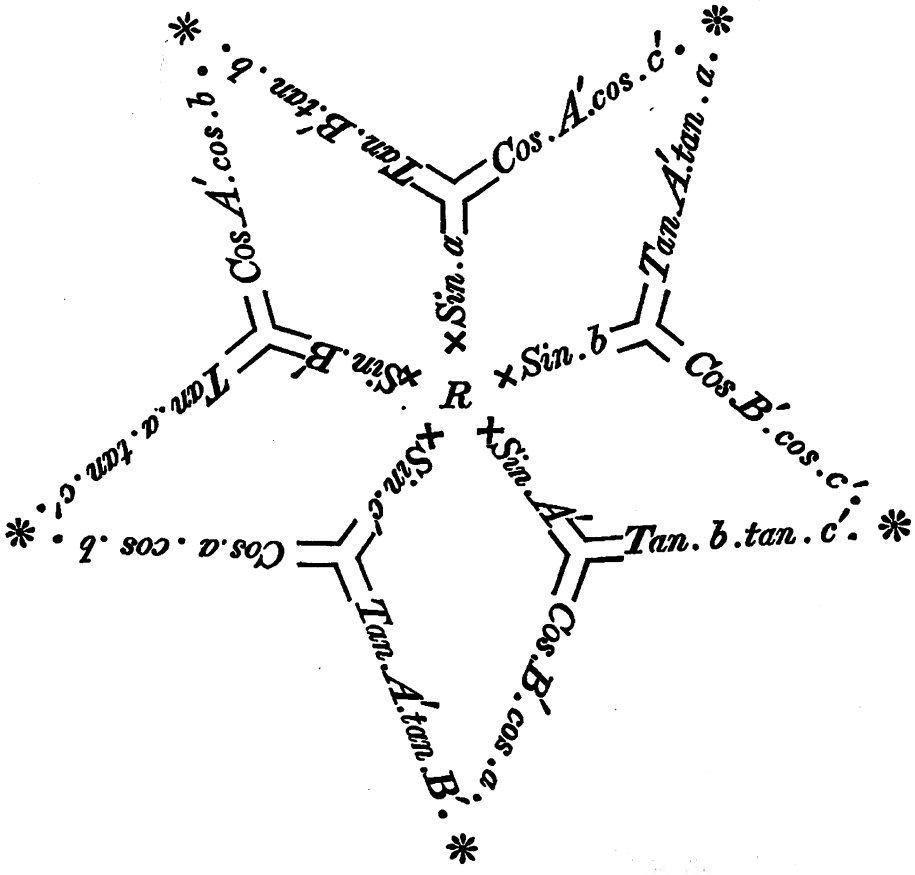
Rad × sin middle part = product of cos opposite parts.

Both these rules may be comprehended in a single expression, thus,

Rad sin mid. = prod. tan adja = prod. cos opp.;

and to retain this in the memory we have only to remember, that the vowels in the contractions *sin*, *tan*, *cos*, are the same as those in the contractions *mid*, *adja*, *opp*, to which they are joined.

These rules comprehend all the succeeding equations, reading from the centre, R = radius.



In the solution of right-angled spherical triangles two parts are given to find a third, therefore, it is necessary, in the application of this formula, to choose for the middle part that which causes the other two to become either adjacent parts or opposite parts.

Examples.

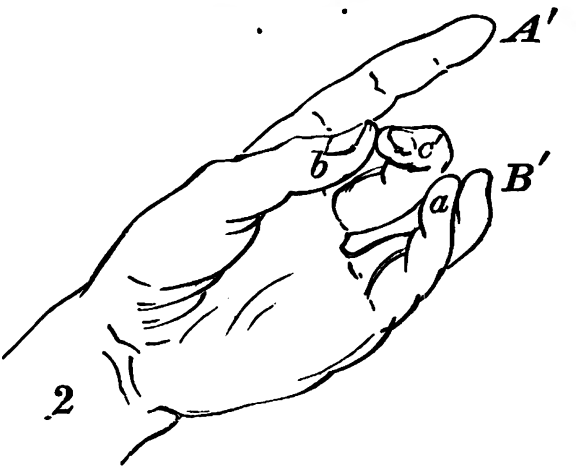
1. In a right-angled spherical triangle, the hypothenuse

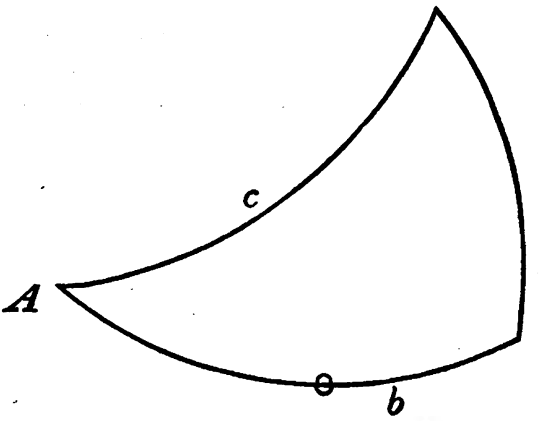
$c = 61°\ 4'\ 56''$; and the angle
$A = 61°\ 50'\ 29''$. Required, the adjacent leg?

	°	′	″	
	90	0	00	
$c =$	61	4	56	
	28	55	04	$= c'$.

	°	′	″	
	90	0	00	
A =	61	50	29	
	28	9	31	= A.

In this example, A′ is selected for the middle part, because then b and c' become adjacent parts, as in the annexed figure.





$$\text{Rad} \times \sin A' = \tan b \times \tan c'.$$

$$\therefore \tan b = \frac{\text{rad} \times \sin A'}{\tan c'}.$$

By Logarithms.

Rad — ..... —	10.0000000
Sin A' — 28° 9′ 31″ —	9.6738628
	19.6738628
Tan c' — 28° 55′ 4″ —	9.7422808
Tan b — 40° 30′ 16″—	9.9315820

The side adjacent to the *given* angle is acute or obtuse, according as the hypothenuse is of the same, or of different species with the *given* angle.

$\therefore$ the leg $b = 40°\ 30'\ 16''$, acute.

Supposing the hypothenuse $c = 113°\ 55'$, and the angle $A = 31°\ 51'$, then the adjacent leg b would be $117°\ 34'$, obtuse.

(To be continued.)

A NEW AND SIMPLE METHOD TO FIND THE PERPENDICULAR HEIGHT OF MOUNTAINS, HEADLANDS, ETC., ABOVE ANY GIVEN DATUM, FROM BAROMETRICAL AND THERMOMETRICAL OBSERVATIONS.

This article was first published in "Silliman's American Journal of Science and Arts," for the quarter ending with December, 1842.

The succeeding has been found the most simple and easy practical rule, for measuring the elevations and depressions of objects by means of the barometer and thermometer. It is necessary that the observers be provided with two portable barometers, each of the same construction, with a vernier or nonius properly adapted to the scale, and a thermometer attached to each, having their bulbs each of the same diameter, nearly, as the diameters of the barometric tubes. One of these barometers must be placed in the shade at the top of the eminence, with a detached thermometer near it; and let the other barometer be placed below in like manner, with a detached thermometer near it. When the thermometers have acquired

the temperature of the air—that is, when the fluid in each becomes stationary—the observers must note down the temperatures shewn by the thermometers, and the heights of the mercurial columns in the barometers.

Then the elevation of the higher barometer above the lower (or the height of the eminence) may be determined by the following

RULE.—Add the allowance found in Table I., for the difference of temperature taken by the attached thermometer, to the logarithm of that height of the barometer which corresponds to the least degree of the thermometer. Then to the logarithm of the difference of the logarithms of the heights of the barometer observed at the higher and lower stations, thus corrected, add the logarithm of the allowance found in Table II. for the mean temperature of the detached thermometer, when increased by the constant number ·92102; this sum will be the logarithm of the required height in fathoms.

OBS.—The first four figures of the logarithms of the heights of the barometer, together with the indices, are to be counted whole numbers, and the numbers taken from Tables I. and II. must always have five places of decimals, though they need not always be used.

Tables I. and II. may be dispensed with, as 456789 answers to a degree of the attached thermometer in Table I., and 0024680 to a degree of the detached in Table II.

TABLE I.

Of the allowance for the difference of the temperatures of the attached thermometer.

Tens.	Units.	Tenths.				
1	0	4	5 6 7	8	9
2	0	9	1 3 5	7	8
3	1	3	7 0 3	6	7
4	1	8	2 7 1	5	6
5	2	2	8 3 9	4	5
6	2	7	4 0 7	3	4
7	3	1	9 7 5	2	3
8	3	6	5 4 3	1	2
9	4	1	1 1 1	0	1

TABLE II.

Of the allowances for the mean temperatures of the detached thermometers.

Hundreds.	Tens.	Units.	Tenths.				
1	0	0	0	2 4	6	8	0
2	0	0	0	4 9	3	6	0
3	0	0	0	7 4	0	4	0
4	0	0	0	9 8	7	2	0
5	0	0	1	2 3	4	0	0
6	0	0	1	4 8	0	8	0
7	0	0	1	7 2	7	6	0
8	0	0	1	9 7	4	4	0
9	0	0	2	2 2	1	2	0

Previous to M. De Luc commencing his experiments on the barometer, it was taught that a mean between the two temperatures shown by the thermometer attached and the height of the mercury in the barometer at two different stations, was sufficient

to determine the perpendicular distance of those stations. But De Luc found by repeated experiments that an additional or detached thermometer was likewise necessary, which has since been confirmed by General Roy, Sir G. Shuckburgh, and others. However, before making further remarks, we shall illustrate the rules and tables just given by practical examples.

Examples.

I. The heights of the barometer at the bottom and top of a hill are 29.862 and 26.137 inches; the attached thermometer at the bottom and top indicates 68° and 63°; also, the detached thermometer at these stations gives 71° and 55° respectively. It is required to find the perpendicular height of the mountain.

Thermometer attached.		Barometer detached.	
Lower station,	68°	Lower station	71°
Higher „	63	Higher „	55
Difference	5		2)126
		Mean	63

Barometer at summit, where attached thermometer indicates least degree, = 26·137, log. 14172·557

From Table I, for 5 units, we have 2·28394

Log. corrected, 14174·84094

Barometer at the base = 29·862, log. 14751·189

Take 14174·84094

Log. 576·34806 = 2·7606848

Then, from Table II, for 6 tens, ·14808

For 3 units, (making in all 63°,) ·00740

Constant, ·92102

Log. 1·07650 = 0·0320140

Height in fathoms = 620·4385, corresponding to log. 2·7926988

II. Wishing to know the perpendicular height of Croaghanmoira mountain in the county of Wicklow, and having two barometers and detached thermometers, which for months before precisely agreed with each other in different states of the air; leaving an assistant on a level with the sea near Arklow, with directions to make accurate observations every fifteen minutes from 3 to 4 o'clock, (our watches being previously regulated,) I proceeded to the top of the mountain, and at the appointed hour commenced observations. The mean results of the five were as follows: the barometer stood at the summit 28.635,

and at the base 30.609 inches; attached thermometer 61° and 65·5°, and detached thermometer 54·5° and 70° respectively. It is required from these data to find the height of the eminence.

Thermometer attached.		Thermometer detached.	
Lower station,	65·5°	Lower station,	70°
Upper "	61	Upper "	54·5
Difference,	4·5		2)124·5
		Mean,	62·25

Barometer at summit, where attached thermometer is least, } =28·635, log. 14568.972

For 4 units, from Table I, we have 1·82715

For 5 tenths, ·22839

Log. corrected for temperature, 14571·02754

Barometer at the sea = 30·609, log. 14858·491

Subtract 14571·02754

Log. 287·46346 =2·4585826

From Table II, we have,

For 6 tens,	14808	making up for 62·25°.
For 2 units,	00494	
For 2 tenths,	00049	
For 5 hundredths,	00012	
Constant,	92102	
Log.	1·07465	=0·0312670

Hence the height in fathoms = 308·926, log. 2·4898496

It may be observed, that this experiment was repeated at different times, and consequently in various atmospheres, yet the result never varied two feet. It should also be remarked here, that this is not the highest point of Croaghan, for in subsequent experiments another place gave 2177 feet. But the highest summit of the Wicklow mountains is on *Lugnaquillia*, near the source of the river Slaney, which is 3043 feet above the level of the sea.

The above rule will be found to give results as accurate as either that of General Roy or of Sir G. Shuckburgh, and can be applied with greater ease. The investigation of this rule will be given in the author's course of mathematics; the present performance is too limited to enter on such an inquiry.

General Roy makes the height in fathoms

$$= [10000l \mp .468d] \times [1 + (f - 32°) \cdot 00245];$$

Sir G. Shuckburgh

$= [10000l \mp .440d] \times [1 + (f - 32°) \cdot 00243]$ fathoms.

Where l = the difference of the logarithms of the heights of the barometer at the two stations; d = the difference of the degrees shown by Fahrenheit's thermometer, attached to the barometer; f = the mean of the two temperatures shown by the detached thermometer, exposed for a few minutes to the open air in the shade, at the two stations. The sign *minus* takes place when the attached thermometer is highest at the lower station, and the sign *plus* when it is lowest at that station. 10000 (log. M — log. m) was the expression formerly given to find the altitude in fathoms; M, m being the heights of the mercury at the base and summit of any eminence.

This formula is very easily applied, and not far from the truth, when an allowance is made for the increase of temperature above 31°, for this is the degree of temperature to which the above formula is calculated.

As air expands very nearly $\frac{1}{435}$th part of its bulk for every degree of heat, and suffers the same contraction for every degree of cold, the following rule was usually given.

Rule.—Observe the height of the mercury at the bottom of the object to be measured, and again at the top, as also the degree of the thermometer at both these situations, and half the sum of these two last may be accounted the mean temperature. Then multiply the difference of the logarithms of the two heights of the barometer by 10000, and correct the result by adding or subtracting so many times its 435th part as the degrees of the mean temperature are more or less than 31°: the last number will be the altitude in fathoms.

We are too apt to say, when two or more phenomena happen together, that one is caused by the other; where all may be governed by some unknown phenomena. The writer of this article agrees with Mr. Pasley, that the philosophy is false which teaches that expansion is caused by heat, for without fire or heat water is expanded as it becomes ice, and air in the air pump *vacuum;* solids require fire as a means, but the expanding cause itself is perfectly distinct from fire. However, when experiments show that certain phenomena increase and decrease together, one may be taken as an index, if not as a function of the other; but great care ought to be taken not to draw general inferences from limited experiments. At some future time we shall explain why the attached and detached thermometers differ, and also show how they may be made to agree, and at present proceed to the more immediate object of the communication. To illustrate the rule just given, we shall add another example.

Ex. III. If the heights of the barometer at the bottom and

top of a hill are 29·37 and 26·59 inches respectively, and the mean temperature 26°, what is the height ?

Log. 29·37 = 1·4679039
Log. 26·59 = 1·4247183

0·0431856

And ·0431856 × 10000 = 431·856.

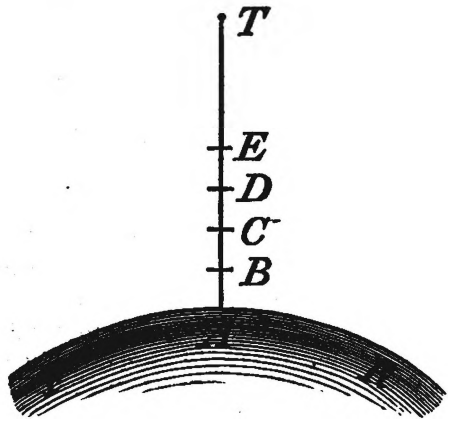
Then 31° — 26° = 5° = the mean temperature below 31°.

∴ 431·856 × $\frac{1}{435}$ 5° = 431·856 × $\frac{5}{435}$ = 4·964, to be subtracted.

Height, 431·856 — 4·964 = 426·892 fathoms.

We shall now investigate the last formula, to give an outline of the theory upon which this proposition is founded.

Let FAR represent part of the surface of the earth, and AT a column of the atmosphere. Conceive this column to be divided into a number of equal and infinitely small parts, as AB, BC, CD, &c., in each of which we may suppose the density to be uniform, because they are infinitely small. Now since the density of the air is always directly as the compressing force, therefore we have the density of the air in any of the portions AB, BC, &c. as the weight of the column of the atmosphere above that place ; that is, if P represents generally the pressure, D the density of any place, P′ the pressure at any



other place, and D′ its corresponding density, we shall have P : D :: P′ : D′ ; that is, the pressure is to the density in a constant ratio, and may be represented by n : 1 ;

∴ P : D :: P′ :: n : 1.

Consequently, D = $\frac{1}{n}$P,
D′ = $\frac{1}{n}$P′,
D″ = $\frac{1}{n}$P″, &c.

That is, the density at any place is equal to, or may be measured by the $\frac{1}{n}$th of the compressing force.

Hence, if we make P stand for the pressure at the surface A, and let each of the parts, A B, B C, C D, &c., be equal 1, then will $\frac{1}{n}$P represent the weight or pressure of the part A B, and

$$\therefore P - \tfrac{1}{n}P = \frac{n-1}{n} P = \text{the pressure at B,}$$

$$\text{and } \frac{n-1}{n^2} P = \text{the density or weight of B C.}$$

In the same way, $\frac{(n-1)^2}{n^2} P =$ the pressure at C,

$$\frac{(n-1)^3}{n^3} P = \text{the pressure at D, &c., &c.}$$

So that the pressure, and consequently the density, will decrease in a geometrical progression, as the altitudes increase in an arithmetical progression.

Calling the density at the surface d^n, and the several altitudes 1, 2, 3, 4, &c., we shall have the following corresponding series :—

Altitudes, 0, 1, 2, 3, 4, 5, &c.
Corresponding densities, d^n, d^{n-1}, d^{n-2}, d^{n-3}, d^{n-4}, d^{n-5}, &c.

Dividing the latter series by d^n, we have

Altitudes 0, 1, 2, 3, 4, 5, &c.
Corresponding densities, 1, d^{-1}, d^{-2}, d^{-3}, d^{-4}, d^{-5}, &c.

This is strictly analogous to the property of logarithms. In fact, the several altitudes form a peculiar system of logarithms, of which the reciprocals of the corresponding densities are the natural numbers ; from this circumstance they have been denominated atmospheric logarithms. From a similar circumstance, the Napierian are termed hyperbolic logarithms, because they express the areas contained between the asymptote and curve of an hyperbola. We shall write these atmospheric logarithms with large letters—thus, "Log."—to distinguish them from the Briggean or common logarithms, which are written "log.$_{10}$" or simply "log.," and also from the hyperbolic, which are denoted by "log.$_n$"

Let a, A represent any two altitudes, and d, D their corresponding densities.

Then will A = — Log. D,
and a = — Log. d;
$\therefore$ A — a = Log. d — Log. D = Log. $\frac{d}{D}$.

Now it is a well known property in logarithms, that by assuming different values for the base, there will be as many different systems of logarithms ; and it is equally well known,

that in all the various systems of logarithms, the logarithms of the same numbers can be converted from one system to another by a constant multiplier or *modulus*.

The object of our present inquiry is to determine a constant multiplier, that shall convert the common logarithm of a number into the atmospheric logarithm of the same number. To accomplish this, let

$$\text{Log.}\frac{d}{\text{D}} = x \text{ log.}\frac{d}{\text{D}},$$

$$\therefore \text{A} - a = x \text{ lon.}\frac{d}{\text{D}}.$$

Then making $a = 0$, or which is the same, if we suppose d to represent the density of the atmosphere at the surface of the earth, we shall have

$$\text{A} = x \text{ log.}\frac{d}{\text{D}}.$$

In order to find x, let us take the height of a homogeneous atmosphere, when the temperature shown by the thermometer is 31°, and the height of the barometer 29½ inches, at 26057 feet; then the density at the surface, and one foot above it, will be

$$a=0\,;\ d=26057.$$
$$\text{A}=1\,;\ \text{D}=26056.$$

That is, the pressure at the surface will be equal to a column of air of uniform density of 26057 feet high; and, consequently, one foot above the surface = 26056 feet high, or a foot less.

Since the densities are as the pressures, we have

$$\text{A}-a=1=x \text{ log.}\frac{26057}{26056};$$

and making $26057=n$,

we have $1=x \text{ log.}\dfrac{n}{n-1}$.

$$\text{But log.}\frac{n}{n-1} = \text{M}\left(\frac{1}{n}+\frac{1}{2n^2}+\frac{1}{3n^3}+\frac{1}{4n^4}+\text{\&c.}\right)$$

And when, as in the present case, n is a large number, all the terms but the first may be neglected as unimportant; also since M = .43429448, the modulus of decimal or common logarithms,

$$\therefore 1 = x \times \frac{\cdot 43429448}{26057};$$

$$\text{whence } x = \frac{26057}{\cdot 43429448} = 60000, \text{ very nearly.}$$

The above formula is reduced to

$$A = 60000 \log. \frac{d}{D} \text{ feet};$$

or putting m and M the height of the mercury at the earth's surface and at the altitude A, then the fraction

$$\frac{d}{D} = \frac{m}{M}.$$

Also since six feet are equal to one fathom, the simple multiplier 60000 for feet becomes 10000 for fathoms, which is more convenient.

Hence, instead of $A = 60000 \log. \frac{d}{D}$ feet,

$$\text{we have } A = 10000 \log. \frac{m}{M} \text{ fathoms,}$$

which is the formula formerly used in measuring altitudes by the barometer.

With respect to the height taken for the homogeneous column of air different writers vary, but this difference does not affect the ultimate result. It is well established, that the height of an homogeneous atmosphere whose density would be equal to that of the air at the earth's surface, and weight the same as that of the real atmosphere when compared with a column of mercury, or any other fluid, of the same weight, the heights will be reciprocally as the specific gravities of the air and mercury or other fluid. So that, if we take the specific gravity of the air at the earth's surface at $1\frac{1}{5}$ when compared with distilled water at 1000, and that of mercury at 14000, also the column of mercury in the barometer $= 29\frac{1}{2}$ inches, we have

$1\frac{1}{5} : 14000 :: 29\frac{1}{2} : 344166$ inches $= 28680.5$ feet $= 5.43$ miles.

But the specific gravity of fluids varies as their temperatures vary. It has been found by various experiments, that when the mercury in the barometer stands at 30 inches, and the thermometer at 55°, the specific gravity of air, water, and mercury are nearly as $1\frac{1}{5}$, 1000 and 13600. Hence,

$1\frac{1}{5} : 13600 :: 30 : 340000$ inches $= 28333\frac{1}{3}$ feet $= 5.366$ miles,

the height of a homogeneous atmosphere. Again, taking the specific gravity of the air at the earth's surface at $1\frac{2}{3}$, which some affirm, and the barometer at $29\frac{1}{2}$ inches, it will be

$1\frac{2}{9} : 13600 :: 29\frac{1}{2} : 328255$ inches $= 27358$ feet $= 5.1814$ miles.

Hence, generally, we may assume if the air was of the same density at all altitudes as at the earth's surface, its height would be between five and six miles. But it matters not what degree of temperature we assume, for we can always accommo-

date the result to any other temperature, as before observed, by augmenting or diminishing the result by the $\frac{1}{435}$th part for every degree above or below 31°.

It may further be observed, that the common barometer is the best and most to be depended on; for many which are said to be improved, have only the recommendation of deviating from it in simplicity. It appears from accurate observations, that mercury stands higher in tubes of a larger than in those of a narrower bore; and therefore, when observations are made with different barometers, attention should be paid to the difference of their diameters. In order to prevent the effects of the attraction of cohesion, the bore of the tube should not be less than one-fourth of an inch; but one third of an inch would be better.

We cannot conclude this problem better than by giving a remark of Prof. Dalby's, whose practice in trigonometrical surveying was extensive.

"In determining altitudes by the barometer, it is best to make the observations at the upper and lower stations at one and the same time as nearly as possible; but great care must be taken that the two barometers and also the thermometers are alike; that is, they should precisely agree when together in all states of the air. It is also necessary that the specific gravity of the mercury be well ascertained, because it is not equally pure in all barometers, which is the principal reason why different results are frequently obtained from observations made with different barometers at the same stations. Other circumstances, however, not generally known, may contribute to such disagreement."

Thus Mr. Ramsden proved by experiment, that the quick-silver in barometer tubes made of different sorts of glass, will be sustained at different heights.*

If several sets of observations be made at each station, after short intervals of time, and the mean of the separate results be taken, the conclusion will probably be more accurate than that derived from a single observation.

* It is usual at present, in accurate meteorological records, to note the difference of height between barometers of flint and hard glass. (See the Tables of the Royal Society.)

MEASUREMENT OF ANGLES.

From the "Civil Engineer and Architect's Journal," for Oct. and Nov. 1847

A New Method of Measuring the Degrees, Minutes, &c., in any Rectilinear Angle, by Compasses only, without using Scale or Protractor.

LET it be required to find the number of degrees, minutes, &c., in the angle A C B = θ (Fig. 1). With any radius A C, describe a circle: then

Fig. 1.

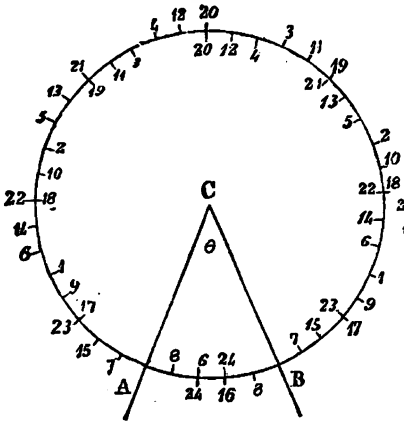
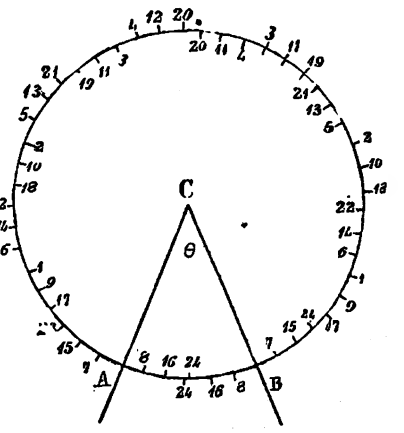


Fig. 2.



take A B in the compasses, and apply it from B to 1; from 1 to 2; from 2 to 3; &c. (the numbers outside the circle are referred to). If, in applying the arc A B, we find that on our return to B, after n applications we have a coincidence, then it is well known that the number of degrees, &c. will be $= \frac{360}{n}$. But, in the present example, after eight applications the point falls at 8, putting Δ_1 = from 8 to B, continue to apply the same arc or opening of the compasses from 8 to 9; from 9 to 10; from 10 to 11; &c., on to 16. This process is to be continued till we have the half or more than half the arc A B between the last point found and B. In this case 24 is the point. Any error that may be involved in the process will be much neutralized by thus determining the points 8, 16, 24, &c. independently. Theoretically, the arcs B, 8; 8, 16; 16, 24, &c. are all equal, but practically they may imperceptibly differ.

We might have taken the arc B, 8, and applied it from 8 to 16; from 16 to 24, &c.; but this process would multiply any error that might be involved in B, 8; while the process just described has a correcting tendency. To lessen error further, we are again to begin at A, and apply the arc A B in a contrary direction, from A to 1; from 1 to 2; from 2 to 3, &c. (the numbers inside the circle are in this case referred to). Should the points 24 and 16 coincide, as in fig. 1, then we have

$$8\,\theta + \Delta_1 = 360°;\ \text{and}\ 5\,\Delta_1 = \theta;$$

$$\therefore\ \Delta_1 = 360° - 8\,\theta = \frac{\theta}{5};\ \therefore\ 1800 - 40\,\theta = \theta;$$

$$\therefore\ \theta = \frac{1800}{41} = 43°\ 54'\ \tfrac{6}{41}.$$

If the points 24, 24, overwrap or fall as in fig. 2.—Then put Δ_2 = from 24 to 24: this arc will be very small in most cases —in this case it is the 20th part of A B;

$$\therefore\ \theta = 20\,\Delta_2;\ 8\theta + \Delta_1 = 360°;\ 6\,\Delta_1 - \Delta_2 = \theta.$$

From these equations, which involve the unknown quantities θ, Δ_1, Δ_2 θ is really eliminated.

$$\Delta_1 = 360° - 8\,\theta,\ \text{from the second};$$

$$\text{and}\ \Delta_1 = \frac{\theta + \Delta_2}{6},\ \text{from the third.}$$

$$\therefore\ 2160 - 48\,\theta = \theta + \Delta_2 = \theta + \frac{\theta}{20};\ \text{since}\ \Delta_2 = \frac{\theta}{20}.$$

$$\therefore\ \theta = \frac{43200}{981} = 44°\ 2'\ \tfrac{22}{109}.$$

If the points 24, 24, do not overwrap, as in fig. 3, and Δ_1 be in excess instead of defect, that is, that some multiple of θ made less by Δ_1 make up the circumference. In this case the three equations will stand thus:—

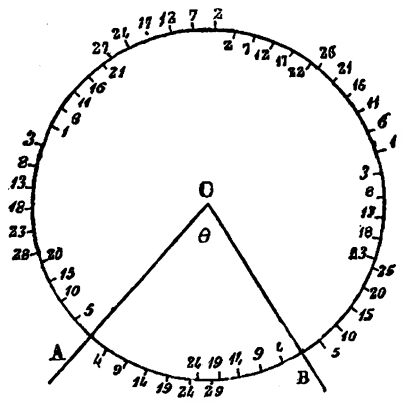
$$5\,\theta - \Delta_1 = 360°;\ 10\,\Delta_1 + \Delta_2 = \theta;\ \text{and}\ 29\,\Delta_2 = \theta.$$

In this example, the distance between 24 and 24, or Δ_2, is found to be the 29th part of the arc A B.

$$\therefore\ \theta = \frac{52200}{711} = 73°25'\ \text{nearly.}$$

It is evident that the numbers on these figures may be omitted in practice, as none of them except the first is required; indeed, where the points of the compasses rest need not be noted, except those points that fall inside the points A, B.

Fig. 3.



This method of measuring an angle is more accurate and expeditious than may at first appear from the above lengthened details, and will often be found convenient when compasses only can be obtained. A general rule may be arrived at as follows: Let

$$m\,\theta \pm \Delta_1 = \pi = 360^\circ\,;\; n\,\Delta_1 \pm \Delta_2 = \theta\,;\text{ and }\; q\,\Delta_2 = \theta\,;$$

be the three equations generally expressed.

$$\theta = \frac{n\,q\,\pi}{m\,n\,q \pm (q \mp 1)}\,. \qquad (Q)$$

In example, fig. 2, this expression becomes

$$\theta = \frac{6 \times 20 \times 360}{8 \times 6 \times 20 + 20 + 1} = 44^\circ 2' \cdot 201835.$$

In example, fig. 3,

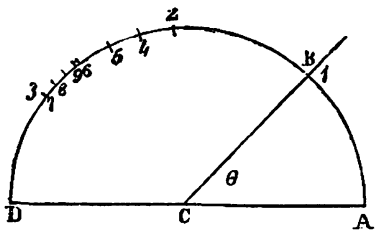
$$\theta = \frac{10 \times 29 \times 360}{5 \times 10 \times 29 - 29 + 1} = 73^\circ 25' \cdot 063.$$

The only thing to be observed in (Q) is the sign of $q \mp 1$. In examples like the latter it is to be minus, but in those like the former plus.

This method of measuring angles will be found more correct than the ingenious one proposed by M. De Lagny, which consists in measuring angles with a pair of compasses, and that too without any scale whatever, except an undivided semicircle. Having any angle drawn upon paper, to measure it: produce one of the sides of the angle backwards behind the angular point; then with a pair of fine compasses describe a pretty large semicircle from the angular point as centre, cutting the sides of the proposed angle, which will intercept a part of the semicircle. Then take this intercepted part very exactly

between the points of the compasses, and turn them successively over upon the arc of the semicircle, to find how often it is contained in it, after which there is commonly some remainder; then take this remainder in the compasses, and in like manner find how often it is contained in the last of the integral parts of the first arc, which will again most likely give some remainder; find in like manner how often this last remainder is contained in the former; and so on continually, till the remainder becomes too small to be taken and applied as a measure. By this means M. De Lagny obtained a series of quotients, or fractional parts one of another, which being properly reduced into one fraction, give the ratio of the first arc to that of a semicircle; or the ratio of the proposed angle to two right angles or 180 degrees, and consequently the degrees and minutes of the angle itself becomes known.

Fig. 4.



Suppose the angle A C B (fig. 4) be proposed to be measured. Produce A C towards D; and from the centre C, describe the semicircle A B D, on which A B is the measure of the proposed angle. Take A B in the compasses, and apply it three times on the semicircle, at 1, 2, and 3; then take the remainder D, 3, and apply it back upon 3, 2, which is but once, namely, at 4; again, take the remainder 4, 2, and apply it three times on 4, 3, at 5, 6, and 7; then take 3, 7, and apply it twice on 7, 6, at 8, and 9; lastly, take the remainder 9, 6, and it will be found to be contained just five times, in 8, 9. Hence the series of quotients in this particular example is 3, 1, 3, 2, 5, which give the continued fraction,

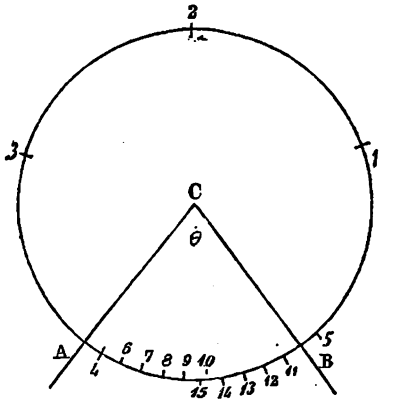
$$\tfrac{1}{3}+\tfrac{1}{1}+\tfrac{1}{3}+\tfrac{1}{2}+\tfrac{1}{5},$$

which, when properly reduced, gives vulgar fraction $\frac{49}{185}$, and $\frac{49}{185}$ of $180^\circ = 47^\circ\ 44'$ nearly.

To those acquainted with the doctrine of continued fractions this method of De Lagny is easy enough, and very accurate considering the means employed. If great accuracy be not

required, our method may be much contracted, by only applying the arc once round the circle, and then using Δ_1 to find all the other required numbers. Taking the same angle as the one

Fig. 5.



measured in example 3, apply A B (fig. 5), from B to 1 ; from 1 to 2 ; from 2 to 3 ; from 3 to 4 ; from 4 to 5. Then take B, 5, in the compasses, and apply it from B to 11 ; from 11 to 12 ; from 12 to 13 ; from 13 to 14 ; and from 14 to 15, near the middle of the arc A B. With the same opening B, 5, or A, 4, or Δ_1, as we have termed it, lay off, 4, 6 ; 6, 7 ; 7, 8 ; 8, 9 ; and 9, 10. Then the arc between the points 15 and 10 is found to be contained 33 times in the arc A B ; but before it was contained 29 times, for 29 Δ_2 was found equal to θ. But by this latter contracted process we find that 33 Δ_2 is equal θ. Our object is to show that this discrepancy will not alter in any great amount the result or measure of the angle in degrees, minutes, &c. From (Q) we have

$$\theta = \frac{nq\pi}{mnq \pm (q \mp p)} = \frac{10 \times 33 \times 360}{5 \times 10 \times 33 - 33 + 1} = 73^\circ\, 25'\, \tfrac{355}{809}.$$

This result was $73^\circ\, 25'\, \frac{5}{79}$. when q was 29.

To obtain the divisor of (Q), the three numbers m, n, q, have to be multiplied together ; to their product q is to be added if m be too small, but subtracted if too great ; to this sum or difference we must add one if n be too great, but subtract one if n be too small. In the latter case, (Q) becomes $\frac{10 \times 33 \times 360}{5 \times 10 \times 33 - 33 - 1} = 73^\circ\, 30'$ nearly, a result which differs from the former results only by 4 or 5 minutes. This circumstance points out the great value of the rule, for it is evident that the result remains nearly the same, whatever be the positions of the points between A and B. Or in other words, the care-

lessness of the operator does not much affect the result, for in all cases it comes nearly right. I must digress, and add,—what a pity that our statesmen, architects, engineers, &c., cannot discover a few rules of this kind.

MEASUREMENT OF ANGLES.

How to lay off an Angle of any number of Degrees, Minutes, &c., with Compasses only, without the use of Scale or Protractor.

LET it be required to lay off an angle of $36° 40' = \beta$.—Take any small opening of the compasses less than one-tenth of the radius, and lay off any number of equal small arcs, from A to 1; from 1 to 2; from 2 to 3; &c. (fig. 1), until we have laid off an arc, A B, greater than the one required.

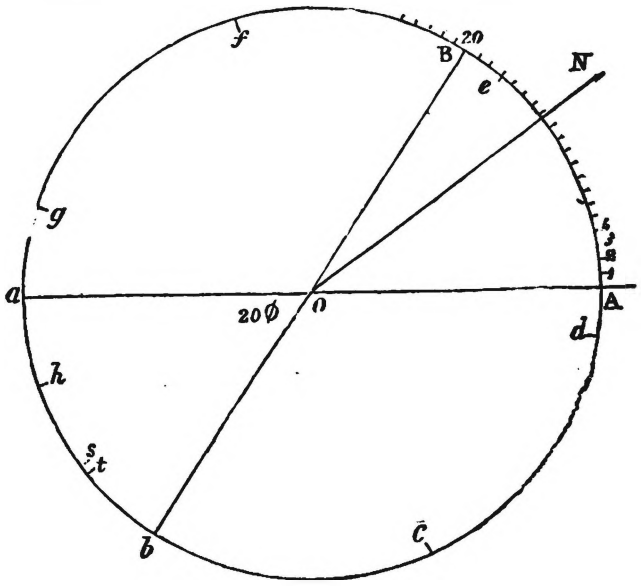


Fig. 1.

Draw B b through the centre o, then will the arc $a\,b$ = arc A B, which we shall put $= 20\,\phi$ in this example, and proceed to measure $a\,b$ as in example fig. 5, page 61. Lay off $a\,b$ from b to c; from c to d; from d to e; from e to f; from f to g. Putting $g\,a = \Delta_1$, then

$$6 \times 20\,\phi + \Delta_1 = 360° = \frac{108}{11}\beta\,;\ \text{because}$$

$$\frac{360°}{36°\,40'} = \frac{21600}{2200} = \frac{108}{11}.$$

Lay off, as before directed, $g\,a, = \Delta_1$, from a to h, from h to s, and b to t; then calling $s\,t$, Δ_2, we have

$$3\,\Delta_1 + \Delta_2 = 20\,\phi\,;$$

and we find that $s\,t$ is contained 28 times in the arc $a\,b$;

$$\therefore 120\phi + \Delta_1 = \frac{108}{11}\beta\,;\ 3\,\Delta_1 + \Delta_2 = 20\,\phi\,;\ \text{and } 28\,\Delta_2 = 20\,\phi.$$

Eliminating Δ_1 and Δ_2, we find

$$\beta = \frac{29205}{2268}\phi = 12{\cdot}9 \text{ times } \phi \text{ nearly};$$

$\therefore$ $36^\circ\,40' = \angle\,\mathrm{A}\,o\,\mathrm{N}$ is laid off with as much ease and certainty as by a protractor.

As a second example, let it be required to lay off an angle of $132^\circ\,27'$.—From $180^\circ\,0'$ take $132^\circ\,27' = 47^\circ\,33'$, which put $= \beta$.

$$\frac{360^\circ}{47^\circ\,33'} = \frac{2400}{317} \text{ when put} = \frac{\nu}{\delta},\ \text{then } \frac{\nu}{\delta}\beta = 360^\circ = \pi.$$

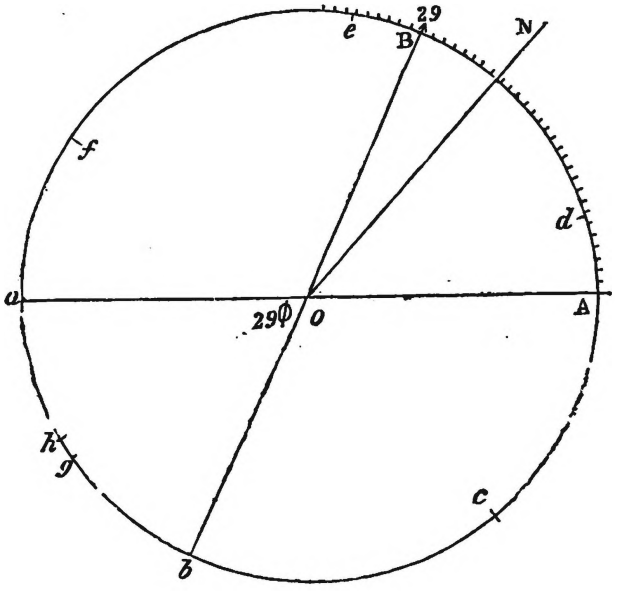


Fig. 2.

Referring to fig. 2, we have laid off 29 small arcs from A to B; $29 = \epsilon$. $\mathrm{A\,B} = a\,b = b\,c = c\,d = d\,e = e\,f$. And $a\,g = b\,h = a\,f = \Delta_1$; $h\,g = \Delta_2$.

$$\therefore 5 \times 29\,\phi + \Delta_1 = 360^\circ = \frac{\nu}{\delta}\beta = m\,e\,\phi \pm \Delta_1 \qquad (1)$$

$$2\,\Delta_1 - \Delta_2 = 29\,\phi,\ \text{or } n\,\Delta_1 \pm \Delta_2 = \epsilon\,\phi \qquad (2)$$

$$13\,\Delta_2 = 29\,\phi,\quad \text{or} \quad q\,\Delta_2 = \epsilon\,\phi \qquad (3)$$

Eliminating Δ_1 and Δ_2, we have

$$\beta = \frac{\{mnq \pm (q \mp 1)\}\epsilon\delta}{\nu nq}\phi = \frac{\{5.2.13+(13+1)\}29.317}{2400.2.13}\phi =$$

$\frac{1323729}{62400}\phi = 21\frac{1}{4}$ times ϕ very nearly. Hence the line o N determines the angle $a\ o\ \mathrm{N} = 132° 27'$.

In the expression

$$\beta = \frac{\{mnq \pm (q \mp 1)\}\epsilon\delta}{\nu nq}\phi \quad \text{(R)}$$

substituting the numerals of the first example, then

$$\beta = \frac{\{6.3.28+(28-1)\}\ 20.11}{108.3.28}\phi =$$

$\frac{29205}{2268}\phi = 12{\cdot}9$ times ϕ nearly, the result before obtained.

The ambiguous signs of (R) cannot be mistaken or lead to error, if the manner in which it is deduced from (1), (2), (3), be attended to. From (3)

$\Delta_2 = \frac{\epsilon\phi}{q}$; substituting this value of Δ_2 in (2),

$n\Delta_1 = \epsilon\phi \mp \Delta_2 = \epsilon\phi \mp \frac{\epsilon\phi}{q}$; which, when substituted for Δ_1 in (1), gives

$\frac{\nu}{\delta}\beta = m\epsilon\phi \pm \frac{1}{n}\left(\epsilon\phi \mp \frac{\epsilon\phi}{q}\right)$; from which (R) is found.

OF LOGARITHMS.

From the "Civil Engineer and Architects' Journal," Aug. 1847.

SIR,—Having known for years the readiness with which you publish any thing interesting in art or science, even when it is not in strict accordance with the avowed objects of your excellent Journal, I take the liberty of sending you a few remarks on the construction of logarithms. Indeed, I know of no other periodical open to mathematical communications, particularly when the subjects require woodcuts to illustrate, or symbolical language to investigate.

Logarithms is as powerful an agent in calculation as steam is in mechanics; with this truth before us, it is strange that few know their proper use, or how they are computed,—and fewer still, from the great labour attending the operations by any known method, attempt the calculation of these very important

numbers. Since the days of Napier* and Briggs, logarithmotechny, in a practical point of view, has received but little improvement, while logarithmic formulæ have been cultivated with great success, and advantageously employed to abridge many analytical inquiries in different parts of mathematics. However, it is also true, that some analysts have bestowed much time and labour in search of a simple and direct mode of calculating logarithms, and though wholly unsuccessful, or very nearly so, as respects the ostensible object of the inquiry, they have been rewarded by the discovery of those interesting and momentous formulæ which constitute what is at present termed "the Theory of Logarithms." It is also worthy of remark, that Briggs, Halley, Sharp, Vlacq, and others, who brought the doctrine of logarithms to perfection, were not averse to arithmetical calculations; but our modern mathematicians depend by far too much on purely algebraical expressions, foreign translations, and mere hocus pocus operations on operatical symbols.

In an inquiry on logarithms, it is usual to put N = any given number, a = the base of any system, and M = the *modulus* of the system. Substituting $1 + n$ for N, &c., we have

$$\log.(1+n) = \mathrm{M}\left(n - \tfrac{1}{2}n^2 + \tfrac{1}{3}n^3 - \tfrac{1}{4}n^4 + \tfrac{1}{5}n^5 -,\right) \&c.,$$

for the fundamental expression, from which several other formulæ are derived, hitherto used in the computation of logarithms. But the above series is only useful when n is a very small fraction; while the majority of those deduced from it, are only available in the process of determining logarithms from the combinations of others. The value of M, in the above series, cost Mr. Briggs 54 successive extractions of the square root, and 54 multiplications; and although many ingenious contrivances have been devised to abridge the labour of these extractions, the process is at best very tedious.

Lagrange converted the above series into

$$\log. m = r\,\mathrm{M}\left\{(m^{\frac{1}{r}} - 1) - \frac{(m^{\frac{1}{r}} - 1)^2}{2} + \frac{(m^{\frac{1}{r}} - 1)^3}{3} -, \&c.\right\}$$

by substituting $m^{\frac{1}{r}}$ for $1 + n$; r being entirely arbitrary. This formula can be rendered as convergent as we please, and therefore the value of r can be so assumed, that the logarithm of any number, m, can be determined to a limited extent, by using only the first term of the series, viz., from the equation—

$$\log. m = r\,\mathrm{M}\,(m^{\frac{1}{r}} - 1).$$

* It may not be out of place to add, that our English biographers have always attributed the invention of Logarithms to Napier, an assertion which is, however, untrue; a Frenchman of the name of Juste Byrge being the original discoverer, as is clearly proved in the "Biographie Universelle." (See Robert H. O'Byrne's "Representative History of Great Britain and Ireland," part i., p. 75.) Juste Byrge also invented the proportional compasses.

This method, undoubtedly, is always applicable to the direct computation of a logarithm ; yet it is the same in effect as that proposed by Briggs, and is equally laborious, on account of the great number of extractions generally required.

It is, perhaps, unnecessary to dwell at any great length on the difficulties attending the computation of logarithms by a direct process, independently of other logarithms ; however, we cannot conclude these remarks without giving a remarkable expression, deduced by Professor Wallace, of Edinburgh. The form is this—

$$\log. x = \frac{b^{\frac{z}{m}}}{x^{\frac{z}{n}}} \cdot \frac{n\,(x^{\frac{1}{n}} - 1)}{m\,(b^{\frac{1}{m}} - 1)} \; ;$$

in which m and n are any numbers chosen at pleasure; z, always some value between 0 and 1 ; and b, the given base of the system. This expression leaves the base unrestricted, involves no infinite quantity, and is said by some to be "of great analytical elegance ;"—yet, it is purely algebraical, and as to its practical utility in the actual determination of a logarithm, it is just as much use as any other intelligible hieroglyphics.

Perhaps you will allow me to state a fact, which you have tested *—*i.e.* that I have discovered a method by which the logarithm of any number, to almost any extent, may be calculated, independently of other logarithms, in a few minutes. Mathematicians and the curious will, I have no doubt, be obliged to you for publishing the following results. It is well known that when the diameter of a circle is *one*, the circumference is

3·14159265358979323846264338327950288419716939937511,

to 50 places of decimals. Now, I find the logarithm of this number to be

·49714987269413385435126828829089887365167832438044,

which is true to 50 places. For the information of the general reader, it may be necessary to mention, that the logarithm of a number consisting of so many places of figures, has not been before computed to anything near this extent; for, by any of the known methods, such a calculation is almost impossible. From the above result, the logarithm of the area of a circle, when the diameter is *unity*, may be readily deduced, and is found to be

$\bar{1}$·89508988136617146392379049884191282011529856145642 ;

correct to the last figure.

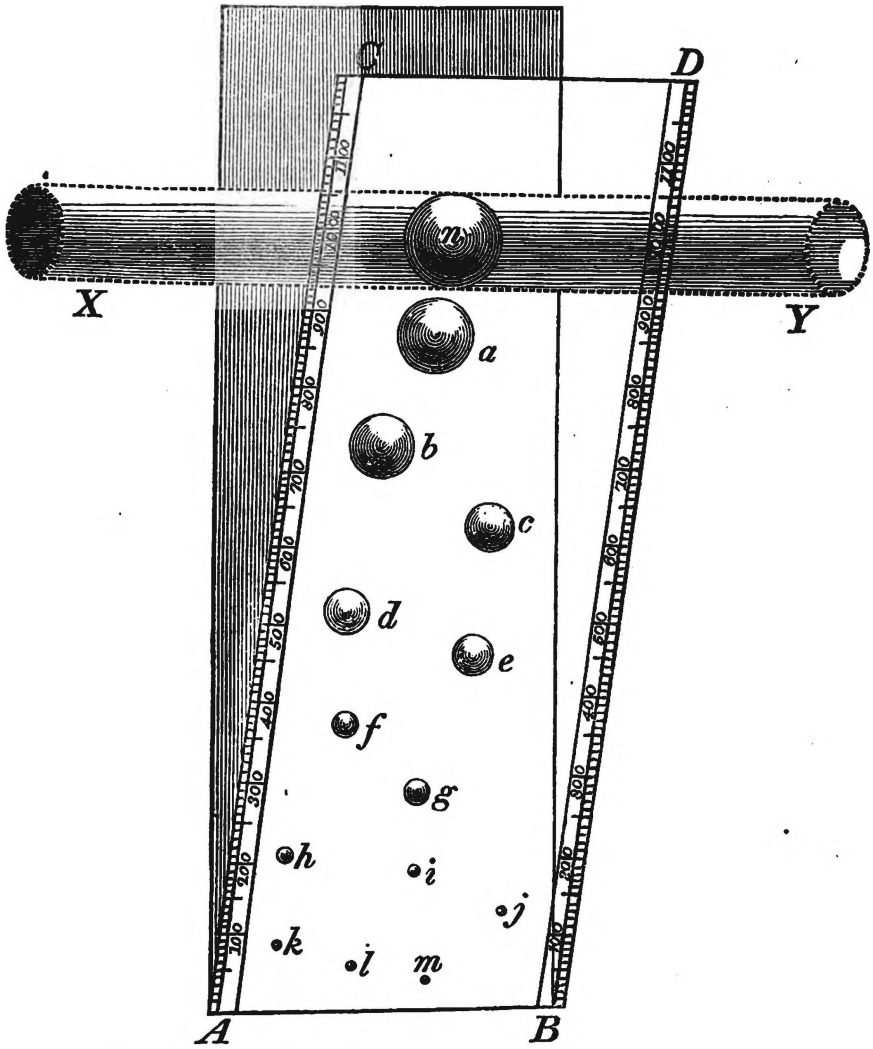
With equal facility, we obtain the logarithm of the contents of a sphere, when the diameter is *unity*, to be

* [We have witnessed Mr. Byrne's facility in calculating logarithms without the use of any book. It is highly desirable that his system of calculation should be revealed to the public.] Ed. C. E. & A. Journal.

$\bar{1}\cdot 7189986223104902218425014902112905376833595787276 4.$
$M = \cdot 4342944819032518276511289189166050822943970058036666.$

July, 1847. O. B.

A SIMPLE METHOD OF FINDING THE WEIGHT OF ANY VERY MINUTE METALLIC GLOBULES WITHOUT A BALANCE.*



* Written for the English Translation of Plattner's excellent Work on the Blow-pipe. Communicated to "The Chemist" by the Author. June, 1844.

LET A B C D, and A B F G, be two pieces of very finely polished plate glass, joined together at A B, forming with each other a very acute angle.

The figure is distorted, and the necessary mounting omitted, for the purpose of exhibiting the parts more distinctly. A C and B D represent scales of equal parts, of a greater or less degree of fineness, according to the exactness required; they may be drawn on paper, and pasted to the glass. X Y is a cylindrical piece of wire of any convenient length, or of any sort of metal, placed between the plates of glass, near the top, parallel to A B: the planes are tangent planes to the cylindrical wire. This wire must have a fixed position while experimenting. The diameter of this cylinder can be calculated with very great accuracy, by weighing a pound, or a half-pound, of the wire from which it is cut; this weight divided by the length of X Y will give its weight, and, from knowing the specific gravity of the metal, the diameter of the wire can be thus accurately determined; this diameter will be the diameter of the globe *n*, the greatest possible that can be made out of the cylinder X Y. The solidity of this globe can be also readily ascertained, and would stand in the same position, subtending the same acute angle, if the cylinder were removed.

Suppose the *inch* and the *ounce* to be the units of measure and weight respectively, L the length of a piece of wire weighing q ounces, from which X Y $= l$ is supposed to be taken, then $\frac{q\,l}{\mathrm{L}}$ = the weight of X Y.

Then say as, $\mathrm{S} : \frac{q\,l}{\mathrm{L}} :: c : \frac{c\,q\,l}{\mathrm{L\,S}}$ = the solidity of X Y in cubic inches, S being the specific gravity of the metal, or the numbers of ounces in 1728 ($= c$) cubic inches;—

$\therefore \frac{c\,q}{\mathrm{S\,L}}$ = area of the circular cross section of X Y.

$\pi = 3{\cdot}14159$, &c.

$\sqrt{\frac{4\,q\,c}{\pi\,\mathrm{L\,S}}}$ = the diameter of this section, or of the globe *n*, the solidity of which

$= \frac{\pi}{6}\sqrt{\frac{64\,q^3\,c^3}{\pi^3\,\mathrm{L}^3\,\mathrm{S}^3}} = \frac{4}{3}\sqrt{\frac{q^3\,c^3}{\pi\,\mathrm{L}^3\,\mathrm{S}^3}} = \sqrt{\frac{16\,q^3\,c^3}{9\,\pi\,\mathrm{L}^3\,\mathrm{S}^3}}$ an expression well adapted to logarithmic computation, which is independent of the length of X Y.

Example.—Suppose a piece of copper wire 376·5 inches long, to weigh 5 ounces, what is its diameter? And what is

the solidity of the greatest globe that can be taken out of its cross section ?

Let L = 376·5, S = 9008, c = 1728, q = 5, and π = 3·14159, &c.

Then log. 4. = 0·6020600
log. q. = 0·6989700
log. c. = 3·2375437
Sub. log. π = 9·5028501
Sub. log. L = 7·4242350
Sub. log. S = 6·0453716

Reject 30·... 27·5110304

2) $\bar{3}$·5110304

The diameter of the wire = ·0569528 = $\bar{2}$·7555152
3

$\bar{4}$·2665456

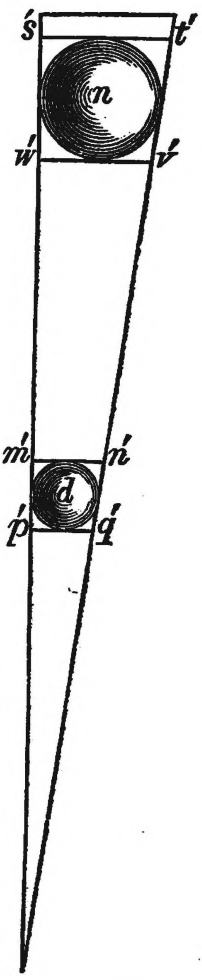
Log. $\frac{\pi}{6}$ = ... $\bar{1}$·7189986

Log. ·00009672622 = $\bar{5}$·9855442

So that the diameter and solidity of a globe, placed in the position of any wire X Y, can be determined to any degree of accuracy required. If the part X Y be taken from such a wire as the one given in the example, the diameter would be ·0569528 inches, and the solidity of n, ·00009672622 cubic inches. The solidity of any other globule, a, b, c, or d, &c., suspended between the planes of glass, can be readily found, from knowing the distance of its centre from the line A B, where the planes meet; and the distance of the axis of the cylinder from the same line. These distances may be measured in a slant line by the scales A C, B D, and will answer as well as the perpendicular distances between the axis and centres. If very great accuracy be required, two distances are necessary—the one above, and the other below the globule: for instance, let us take the globule d, the distance of its lower limb from A B is 49, according to the design of our woodcut, and the distance of the upper limb is 54; therefore its centre may be said to be $\frac{49+54}{2}$ = 51·5 parts from the line AB. It is convenient, but not necessary, to have the wire X Y so placed, that the distance of its axis from A B may correspond with 10, 100, 1000, &c., on the scales A C, B D.

Suppose the wire n to be so placed that s' t' reads on the

scale A C and B D, referring to the first figure at 1002, and w' v' to read 998; then $\frac{1002+998}{2}=1000$, the best position for the wire which keeps the planes apart. Let m' n' be at 645, and p' q' at 643, then the globule may be said to stand at $\frac{645+643}{2}=644$.



Indeed, if the globules were read off all above, or all below, it would answer the same purpose, observing to keep the lines s' t', m' n', or w' v', p' q', &c., parallel.

Then $1000^3 : 644^3 :: S : S_4$; S and S_4 being the solid contents of n and d respectively.

$$\therefore S^4 = \frac{644^3\,S}{1000000000}$$

$\therefore 1000^3 : 644^3 :: W : W_4$; W. and W_4 being the weights of n and d respectively.

$$\text{Hence } W^4 = \frac{644^3\,W}{1000000000}.$$

The weights and solidities, or volumes of the small globules a, b, c, d, e, f, g, h, i, &c., supposing them to stand at 947, 895, 763, 644, 602, 533, &c., respectively, will be in like manner expressed by—

$$a\text{—}W_1 = \frac{947^3}{1000^3}W;\ S_1 = \frac{947^3}{1000^3}S.$$

$$b\text{—}W_2 = \frac{895^3}{1000^3}W;\ S_2 = \frac{895^3}{1000^3}S.$$

$$c\text{—}W_3 = \frac{763^3}{1000^3}W;\ S_3 = \frac{763^3}{1000^3}S.$$

$$d\text{—}W_4 = \frac{644^3}{1000^3}W;\ S_4 = \frac{644^3}{1000^3}S.$$

$$e\text{—}W_5 = \frac{602^3}{1000^3}W;\ S_5 = \frac{602^3}{1000^3}S.$$

&c., &c.

$$\therefore W_1 + W_2 + W_3 + W_4 + \&c., = W\left\{\frac{947^3 + 895^3 + 763^3 + 644^3 + \&c.}{1000000000}\right\};$$

$$\text{And } S_1 + S_2 + S_3 + S_4 + \&c., = S\left\{\frac{947^3 + 895^3 + 763^3 + 644^3 + \&c.}{1000000000}\right\}.$$

The co-efficient, $\left\{\frac{947^3 + 895^3 + 763^3 + 644^3 + \&c.,}{1000000000}\right\}$ or

similar ones, can be determined with great ease by a table of the cubes of numbers; in fact, they may be ascertained by addition. If this co-efficient $= f$, $W_1 + W_2 + W_3 + W_4 +$ &c., $= fw$.

$$\therefore W = \frac{W_1 + W_2 + W_3 + W_4 + \text{\&c.}}{f}$$

It is evident, if the weights W_1, W_2, W_3, &c., of a great number of globules, a, b, c, d, &c., of any sort of metal, be taken together with a very fine balance, and the co-efficient f, determined by a table of cubes, the weight W, of the globe n, can be determined with the greatest possible accuracy, no matter what the specific gravity of the globules may be.

Thus, having found the volume and weight of n—and this may be done to any degree of accuracy—the weight and volume of any other globule or number of globules may be at once determined from—

$$W_1 + W_2 + W_3 + W_4 + \text{\&c.} = W\left\{\frac{947^3 + 895^3 + 763^3 + \text{\&c.}}{1000000000}\right\} = wf.$$

$$S_1 + S_2 + S_3 + S_4 + \text{\&c.} = sf\text{; or}$$

$$\text{Log. } (W_1 + W_2 + W_3 + \text{\&c.}) = \text{log. } w + \text{log. } f.$$

$$\text{Log. } (S_1 + S_2 + S_3 + \text{\&c.}) = \text{log. } s + \text{log. } f.$$

It may be remarked that a detached scale of box or ivory will answer the same purpose as the attached scales A C, B D.

TO FIND THE MAGNITUDE AND FIGURE OF THE EARTH, WITH THE ASSISTANCE OF RAILWAYS;

WITHOUT PRESUPPOSING IT TO BE ANY PARTICULAR FORM.*

To find the length of a degree in any position on the surface of the earth, with the assistance of railroads, presents no obstacle whatever to the mathematician.

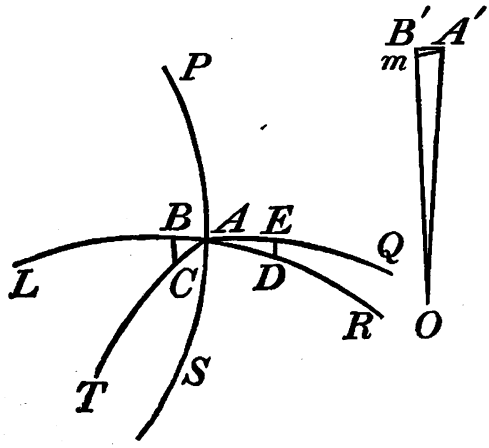
In England, America, and on the Continent, there are at present a considerable number of railroads long enough for the purpose, so that we can have a sufficient number of examples. The lengths, bearings, elevations, and depressions, &c., of the several planes which compose each railroad, may be had from different companies or their respective engineers, and if the dimensions thus obtained are not considered accurate enough, they can be easily taken over again. When this information is acquired, the length of the arc on the surface of the earth between any two given points in the line of road may be very

* This article is extracted from Mr. Byrne's extensive unpublished work, entitled, "A New Theory of the Heavens and Earth."—*Editor of the "Civil Engineer and Architect's Journal."*

readily determined, on account of the approximant of the general line of most railroads to levelness. By astronomical observations the latitudes of both ends of the road, or any point in it may be determined, and also the length of a degree upon any arc passing through either extremity, and a point in the line of road; as the bearings and length of such arc can always be found.

Then supposing a railroad selected of such a length, that from a point in it the length of a degree is known on the arc passing through that point and the beginning of that road, and also that of a degree on the arc passing through the said point and the termination of the road, and the arcs to be oblique; or it will answer the purpose as well to have two roads in the same latitude or nearly so, such that the length of a degree on the arc passing through the extremities of each is ascertained; from which it is required to find the length of a degree on the meridian, and also that of a degree on an arc perpendicular to it, at the same point.

This may be done as follows :—



Let LAR be an arc of the meridian, SAP an arc of the curve at right angles to it, AT one of the arcs upon which the length of a degree is known = L; and AQ the other upon which the length of a degree = l; also, let the bearings of these arcs, or the angles LAT and QAR, which are known = a and b respectively; again, let $g = 57°295779,5$ the arc equal in length to the radius, therefore the radius of curvature at A in the arc AT = Lg; and that at A in AQ = lg. To find the radius of curvature of the arcs RAL and SAP, let the radius of curvature of the meridian at the point A = x, and that of its perpendicular or of the arch SAP at the point A = y. Take AC, an indefinitely small portion of AT = s, and AE, an indefinitely small portion of AQ = s'.

Then $AB = s \cos. a$, and $BC = s \sin. a$, as AC is supposed to be indefinitely small.

Now from the nature of the osculating circle at A, in the arc RAL, the distance of the point B from the horizontal plane passing through A,

$$\text{is } \frac{s^2 \cos.^2 a}{x} :$$

for A′B′, fig. 2, stands in the same position as AB, fig. 1, and may be considered as a perpendicular from the right angle at A or A′, falling between radius of curvature OB′, and the depression mB′ of the point B′ below A′, or that of B below the horizontal plane passing through A ; and the depression of C below B is $= \frac{s^2 \sin.^2 a}{y}$, for the radius of curvature of the arc BC is ultimately the same with that of SAP at the point A, the total depression of C below A is equal,

$$\frac{s^2 \cos.^2 a}{x} + \frac{s^2 \sin.^2 a}{y} = s^2 \left(\frac{\cos.^2 a}{x} + \frac{\sin.^2 a}{y} \right)$$

Again, by the property of the circle above quoted,

$$\frac{s^2}{s^2 \left(\frac{\cos.^2 a}{x} + \frac{\sin.^2 a}{y} \right)} = \text{the radius of curvature of the arc}$$

AT, at the point A $= Lg$;

$$\therefore Lg = \frac{1}{\left(\frac{\cos.^2 a}{x} + \frac{\sin.^2 a}{y} \right)} = \frac{xy}{y \cos.^2 a + x \cos.^2 a} \ldots . \quad (1.)$$

By the same process of reasoning, we find,

$$\frac{xy}{y \cos.^2 h + x \sin.^2 h} = lg \qquad (2.)$$

In (1) and (2) for the sine and cosine of a, substitute s, and c respectively ; and s' and c' for the sine and cosine of h ; let $Lg = r'$, and $lg = r'$. Then equations (1) and (2) become

$$\frac{xy}{y c^2 + x s^2} = r' \qquad (3.)$$

$$\text{and } \frac{xy}{y' c^2 + x' s^2} = r' \qquad (4.)$$

$$y = \frac{r s^2 x}{x - r c^2} \text{ from (3)} \qquad y = \frac{r' s'^2 x}{x - r' c'^2} \text{ from (4)}$$

$$\therefore \frac{r s^2 x}{x - r c^2} = \frac{r' s'^2 x}{x - r' c'^2}, \text{ hence we find}$$

$$x = \frac{s^2 c'^2 - c^2 s'^3}{r s^2 - r' s'^2} \times r r' \text{ and } y = \frac{s'^2 c^2 - c'^2 s^2}{r c^2 - r' c'^2} \times r r.$$

By restoring the values of r and r', we have $x = \frac{s^2 c'^2 - c^2 s'^2}{\mathrm{L}\, s^2 - l s'^2} \times \mathrm{L}\, l g$, and $y = \frac{'s^2 c^2 - c'^2 s^2}{\mathrm{L} c^2 - l c'^2} \times \mathrm{L}\, l g$, the radius of curvature of the meridian, and that of the arc perpendicular at the point A.

$\frac{x}{g}$ = length of a degree on the meridian $= \frac{s' c'^2 - s'^2 c^2}{\mathrm{L}\, s^2 - l s'^2} \times \mathrm{L}\, l.$

$\frac{y}{g}$ = length of degree on arc perpendicular $= \frac{s'^2 c^2 - c'^2 s'^2}{\mathrm{L}\, c^2 - l c^2} \times \mathrm{L}\, l.$

By restoring the values of s, s' c, c', we have $\frac{\sin.^2 a \cos.^2 h - \sin.^2 h \cos.^2 a}{\mathrm{L} \sin.^2 a - l \sin.^2 h} \times \mathrm{L}\, l$, = the length of a degree on the meridian : and $\frac{\text{in.}^2 h \cos.^2 a - \cos.^2 h \sin.^2 a}{\mathrm{L} \cos.^2 a - \mathrm{L} \cos.^2 h} \times \mathrm{L}\, l$ the length of a degree on an arc perpendicular to the meridian at the point A.

By the help of this problem the true figure of the earth, with very little trouble, might be ascertained, as we are not obliged to imagine the earth to be any known form, because be it of whatever form it may, the planes which compose a line of railroad must be in its surface or very nearly so ; nor is it very hard to find a degree or more of road in the same direction.

A SIMPLE METHOD OF COMPUTING THE SPHERICAL EXCESS, WITHOUT THE AID OF LOGARITHMS.

This problem is taken from the " Civil Engineer and Architect's Journal," for July, 1843.

ALTHOUGH Professor Dalby's rule for calculating the *spherical excess* is simple, yet in many instances it is inconvenient, especially when logarithms are not at hand, or when the tables are limited ; moreover, when the radius of curvature at the place of observation is greater or less than $365154{\cdot}6 \times 57{\cdot}2957795$ feet, a more appropriate constant logarithm than that given by Dalby might be used. However, it will be found in practice, that the *spherical excess* can be determined in all cases, by the method here explained, in one half the time usually employed to find the number corresponding to the difference of the logarithms mentioned by Dalby (of which we shall speak hereafter), and therefore the trigonometrical surveyor will in this particular calculation economise a portion of his time.

The following table will be sufficient from 90″ down to $\frac{1}{1000}$

of a second, and can be increased or decreased at pleasure, by multiplying or dividing by 10, if this range be not extensive enough ; the change is easily effected by increasing the number of perpendicular lines to the right and left of those already drawn. Such an alteration may be necessary in speculative inquiries, but can never be required in practice. This method of calculating the *spherical excess*, which depends on the succeeding table, may be better explained by a few examples than by a written explanation.

EXAMPLES.

I. The area of a triangle is 1764724375 square feet ; what is the spherical excess in seconds.

Area	1764724375	
Nearest in the table . .	1697710669	·8·· headed tenths.
Remainder	67013706	
Next nearest in the table	63664150	·3· headed hundredths.
Remainder	3349556	
Next from the table . .	2122138	..1 headed thousandths,
	&c.	831

Therefore the spherical excess = 0″·831.

FOR	Tens of seconds.	Seconds.	Tenths of seconds.	Hundredths of ″	Thousandths of ″
1	2122138	3	3	6	8
2	4244276	6	7	3	6
3	6366415	0	1	0	4
4	8488553	3	4	7	2
5	10610691	6	8	4	0
6	12732830	0	2	0	8
7	14854968	3	5	7	6
8	16977106	6	9	4	4
9	19099245	0	3	1	2

II. Suppose that in the trigonometrical survey of Ireland, under the superintendence of Colonel Colby, the county of Wicklow, which occupies 21986496588 square feet,* to be

* The correct area of Wicklow is 21787766476 square feet ; or, 500178 A. 1 R. 7 P.

taken up with one large triangle, what would be the spherical excess?

	21986496588	Area in square feet.
10·0000 ..	21221383368	which terminate in the column headed tens of seconds, opposite 1.
	765113220	
·3000 ..	636641501	which terminate in the column headed tenths of seconds, opposite 3.
	128471719	
·0600 ..	127328300	answering to six hundredths of a second.
	1143419	
·0005 ..	10610691	there are no thousandths of seconds, but five ten thousandths, found by curtailing another figure.
10″·3605 spl. excess. &c.		

III. The lengths of the chords of a spherical triangle on the surface of the earth are 13, 14, and 15 miles respectively; the area of the triangle formed by these chords is 2341785600 square feet, (which may be taken as the area of the corresponding spherical one, on account of the sides being very small compared with the radius of the earth.) The three angles of the plane triangle are 53° 7′ 48″·358, 59° 29′ 23″·136, and 67° 22′ 48″·506, required the spherical angles.

	2341785600	
1·000	2122138337	for 1 second.
	219647263	
·100	212213833	for $\frac{1}{10}$ of a second.
	7433430	
·003	6366414	for $\frac{3}{1000}$ of a second.

1·103 seconds, the spherical excess.

One third of the spherical excess must be added to each of the plane angles to obtain the spherical angles. Therefore, 1″·103 divided by three = 0″·368 nearly, hence we have,

Plane angles	53° 7′ 48″·358	59° 29′ 23″·136	67° 22′ 48″·506
Third of exs.	·368	·368	·368
	53 7 48·726	59 29 23·504	67 22 48·874

In this example the excess is distributed equally among the angles, because there is no reason to believe one of them to be more erroneous than another. But if one of the angles be suspected to be less correct, or less to be depended on than the others, to this angle must be applied a greater or less portion of the whole correction, according as it is thought to be in excess or defect.

Dalby's Rule.—From the logarithm of the area of the triangle taken as a plane one, in feet, subtract the constant logarithm 9·3267737, and the remainder will be the logarithm of the excess above 180°, in seconds, nearly. By this rule let us work the second example just given :—

Logarithm of 21986496588	...	10·3421561
Constant logarithm	... ...	9·3267737
Logarithm of 10″·3605 ...	...	1·0153824

Dalby's rule, from the simplicity of the wording, and from the few figures exhibited in the operation, appears to be more convenient in practice than the one which we have given as an improvement, but this is not the case; for to find the logarithm of 21986496588, or the number corresponding to log. 1·0153824 from a table of logarithms, will employ more time than the complete working of the problem by the method here proposed, which may be thus investigated.

Let E be the area of a triangle in square feet; γ=the radius of curvature at the place on the earth's surface where the triangle is situated, and A°, B°, C°, be the angles of the triangle considered as spherical, reckoned in *degrees and decimal parts of a degree.* By the well-known theorem of Gerard,

$$2\,E = 4\pi\gamma^2 \left\{ \frac{A^\circ + B^\circ + C^\circ - 180^\circ}{360} \right\} \quad (A)$$

Suppose the triangle to be one employed in the trigonometrical survey of England, then the length of a degree may be considered equal to 60859·1 × 6 feet, without involving much error. Therefore, the circumference of the globe to which this triangle is supposed to belong

$= 360 \times 6 \times 60859{\cdot}1$, and $\gamma = \dfrac{360 \times 6 \times 60859{\cdot}1}{2\pi}$

$$\therefore (A) \text{ becomes, } 2E = 4\pi \left(\frac{360 \times 6 \times 60859{\cdot}1}{2\pi} \right)^2 \left(\frac{A^\circ + B^\circ + C^\circ - 180^\circ}{360} \right)$$

$$\therefore 2E = \frac{360 \times 6^2 \times (60859{\cdot}1)^2}{\pi} (A^\circ + B^\circ + C^\circ - 180^\circ)$$

$\dfrac{2\pi E}{360 \times 6^2 \times (60859{\cdot}1)^2} = A^\circ + B^\circ + C^\circ - 180^\circ$, which we shall call e, or the spherical excess in degrees and decimal parts of a degree, which, when reduced to seconds, will stand thus :—

$$3600\,e = \frac{3600 \times 2\pi E}{360 \times 6^2 \times (60859{\cdot}1)^2} = \frac{\pi E}{1{\cdot}8 \times (60859{\cdot}1)^2}$$

but $\pi = 3{\cdot}1415926$, &c.

$$\therefore \frac{\pi E}{1{\cdot}8\times(60859{\cdot}1)^2}=\frac{E}{\frac{1{\cdot}8\times(60859{\cdot}1)^2}{3{\cdot}1415926}}=\frac{E}{\frac{(182577{\cdot}3)^2}{15{\cdot}707963}} \quad (B)$$

but $\frac{(182577{\cdot}3)^2}{15{\cdot}707963} = 2122138336{\cdot}8$, from whence the rule is derived. It is evident that Dalby's rule is in error when half the length of a degree is greater or less than 182577·3 English feet in the place where the triangle is measured; for by putting (B) into a logarithmic form we have Dalby's rule. But the constant divisor, or its logarithm, can be readily attained by substituting in (B) for 182577·3 feet, half the length of any other degree in feet. However, the difference must be very great when the spherical excess is effected, by such triangles as occur in practice.

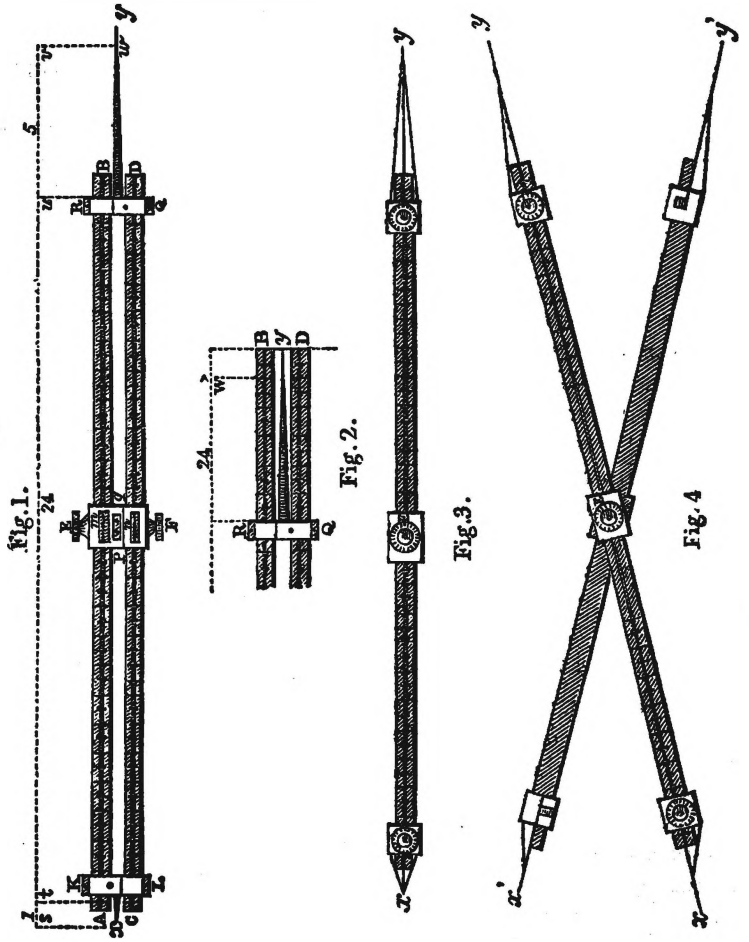
THE IMPROVED PROPORTIONAL COMPASS.

We shall here introduce to the notice of the practical mathematician, an instrument that much facilitates the projection of lines, whether that projection be *Orthographic*, *Stereographic*, or *Gnomonical*. Projections on the perspective plane with this instrument become easy and extremely accurate. The instrument is accompanied by thirty-eight pages of a pamphlet, containing 5,000 tabulated calculations; and as the review of these pages, in "The Civil Engineer," for September, 1846, concisely describes this invention, we give the words of the Reviewer.

"The purposes of this publication are the explanation and illustration of the uses of an instrument invented by Mr. O. Byrne. The object of this invention is to attain increased accuracy in the use of proportional compasses, or rather, to so extend the use of them, that the improved instrument bears to its prototype much the same relation as a Manton to a match-lock.

"With the old proportional compasses, lines could not be reduced with even tolerable accuracy to less than one-ninth their original size; but, by combining with them the vernier scale, the new instrument not only effects much greater reductions with perfect accuracy, but may be made to indicate proportions which extend to several places of decimals. For instance, at page 26 is explained how the compasses may be set to the proportion 1 : 3·1416, the ratio of the diameter of a circle to its circumference. These refined adjustments are perfectly unattainable with the old instrument. Fig. 1 is a side view of Mr. Byrne's invention, of which fig. 3 represents the plan when the instrument is open; fig. 4, when it is shut. Fig. 2 is a

separate view of one end of the compass, *showing how the points can be elongated when worn.*



"The principal feature of the invention is, that the moveable centre E F, about which the two arms turn, is provided with a vernier, and that the points *x*, *y*, are moveable and also provided with verniers. Beside this, there are tightening screws to fix all the points when they have been adjusted.

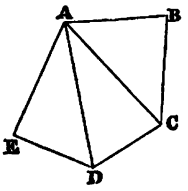
"'The framework of this instrument may be made to assume different proportions and forms for the sake of ornament, compactness, or convenience, according to the fancy or design of the maker; the first completed was constructed by Cary, 181, Strand, London, and explained at York, before the British Association, in 1844. A view of the instrument, which we shall describe, is given (fig. 1); it differs but little from that constructed, according to the directions of the inventor, by Désiré Lebrun, of Paris. The two boxes E *p q*, F *p q*, are so

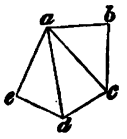
adapted to the beams A B, C D, that they may be moved together by sliding to any part and fixed in that position by tightening the clamp screws E and F. When the moveable centre E *p q* F, is clamped in any position, the instrument turns on an imaginary line or axis passing through E F, perpendicular to a plane passing through P *q*, the junction of the boxes E *p q*, F *p q*, the friction of the planes meeting at *p q*, renders the motion uniform. Connected with the brass boxes at K and L are two points of the instrument meeting at *x*: at K and L there are two clamp screws to fix the boxes, which must be moved together. When the points are in their proper places, the proportions of the instrument that we are describing are these:—Suppose $st = 1$, then $tu = 24$, and $uv = 5$. The points at *x* are represented in proper adjustment, but those at *y* must be moved till *y* comes up the line *v w*, and then made fast by the clamp screws at Q and R.'

"The faces of the arms are graduated to four scales, which mark the proportions for straight lines, circumferences of circles, surfaces, and solids, respectively. Tables of the numbers corresponding to these graduations are given in the publication before us, and as instances of the application of them, we quote the following problems, taken at random:—

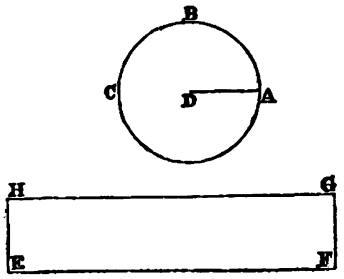
"'In the circumference of a circle A B B' it is required to lay off an arc equal to 3° 39′37″·26.'—Page 20.

"'Let it be required to describe a figure *a b c d e*, similar to A B C D E, whose area will be $\frac{17}{37}$ of it.'—Page 21.



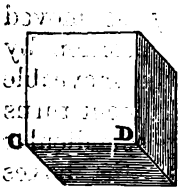


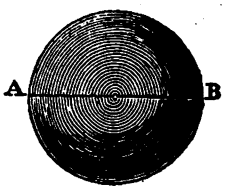
"'Let it be required to construct a parallelogram whose area shall be equal to that of a given circle.'—Page 26.



"'Describe a circle whose circumference will be $11\frac{3}{4}$ inches.'—Page 27.

"'Given the diameter of a globe, to find the side of a cube whose solidity will be equal to that of the globe.'—Page 28.





When the diameter of a globe is 1', the side of a cube equal in solidity to it is ·805996, opposite which, in Table III., 1338 is found, to which if the centre be set, and the diameter of any globe A B taken between the greater opening, the opening of the other points will give C D, the side of a cube whose solidity is equal to that of the globe.

"'In a certain construction the square root of 33 equal parts is required with great accuracy; at what point must the centre be set to effect this object?'—Page 28.

"Mr. Byrne has not spared his labours. The tables which he has given for the purpose of graduating the instrument are very extensive and carried to a greater number of places of decimals than would be generally required in practice. The explanations are so ample and precise that, with this book before him, the draughtsman can never be at a loss respecting the various problems which he is constantly called upon to solve by practical construction. It has been remarked truly, that while the theodolite, sextant, and other instruments of surveying, have been brought to extraordinary perfection by the refinements of modern ingenuity and workmanship, the constructions obtained from these exact instruments are reduced in a very rude manner. While almost everything that could be done for attaining accuracy in the original trigonometrical operations has been effected, the means by which they are rendered practically available have been comparatively disregarded until the invention of the simple instrument which we have been describing."

END OF PART I.

Macintosh, Printer, Great New-street, London.

WORKS BY OLIVER BYRNE.

A TREATISE ON SPHERICAL TRIGONOMETRY. London: published 1835, by John Valpy, M.A. Price 4*s.* 6*d.*

THE NEW and IMPROVED SYSTEM of LOGARITHMS. London: published 1838, by William Day, 17, Gate-street, Lincoln's-inn-fields, where it may be had; and at Bailey's, Cornhill; Currie and Bowman's, Newcastle-upon-Tyne; Bellerby's, York; and at Powell's, Dublin Literary Emporium, 10, Westmoreland-street, Dublin. Price 2*l.* 2*s.*

This "folio" contains a table of the logarithms of the natural numbers from 1 to 10,000,000 (by the help of differences); a Collection of the most useful Trigonometrical Formula; a simple method of converting common logarithms into hyperbolic, and *vice versa*; together with a variety of other particulars relating to the theoretical and practical part of these useful numbers.

A PAMPHLET, showing how to find the Magnitude and Figure of the Earth, with the assistance of Railways, without presupposing it to be any particular form. Newcastle-upon-Tyne: Currie and Bowman, 1838. Price 1*s.*

THE PRACTICAL, COMPLETE, and CORRECT GAGER, containing a description of Byrne's Patent Calculating Instruments; with their Use and Application. Dedicated with permission to Her Most Gracious Majesty Queen Victoria. London: A. H. Bailey and Co., Cornhill. Price 10*s.* 6*d.* 1840.

The principal heads of this Work are, 1. Decimal Arithmetic. 2. The patent sliding Rule and *new Measuring Tape.* 3. The Mensuration of Superficies and Solids. 4. The Gaging of Utensils. 5. Malt Gaging. 6. Cask Gaging. 7. Ullaging. 8. Specific Gravity and the Strength of Spirits. 9. Appendix, on the application of logarithms, equidistant ordinates, &c.

THE DOCTRINE OF PROPORTION, clearly developed, on a comprehensive, original, and very easy system. London: J. Williams, Library of Arts, 106, Great Russell-street, Bloomsbury. Price 7*s.* 6*d.* 1841.

Students who find any difficulty in the fifth book of Euclid, would do well to consult this work. It also contains an Appendix on Incommensurable Magnitudes, and other subjects connected with proportion; but, in no instance is there any vague attempt to *connect number and magnitude.*

THE FALLACIES OF OUR OWN TIME. By Oliver Byrne, and Professor John Byrne, Norfolk, Virginia, United States; author of "An Essay on the Quadrature of the Circle," "Mathematical Disquisitions," &c., &c., &c. London: Sherwood, Gilbert, and Piper, Paternoster-row. 1844.

PART I.—"THE FALLACY OF PHRENOLOGY;" is the London Edition of "Anti-Phrenology," first published at Washington, 1843.

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> Segnius irritant animos demissa per aurem
> Quàm quæ sunt oculis subjecta fidelibus.
>
> HORACE.
>
> A feebler impress through the ear is made,
> Than what is by the faithful eye conveyed.

London: William Pickering, 177, Piccadilly.

Also, early in 1849, will be published, by W. Pickering, with numerous Illustrations, an original work,

ON THE DIFFERENTIAL AND INTEGRAL CALCULUS.

This volume will be uniform with "The Coloured Euclid," but will extend to between 800 and 900 pages, 600 of which are already printed by C. Whittingham, Chiswick. The terms Differential and Integral are retained, although this work develops A NEW SCIENCE, founded upon principles as certain, as simple, and as comprehensive as the elements of plane geometry; consequently, the unsatisfactory illogical doctrines, of the Differential and other methods, are discarded.